Disagreement and Evidence Production in Strategic Information Transmission

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Abstract

We expand Crawford and Sobel's (1982) model of information transmission to allow for the costly provision of 'hard evidence' in addition to conventional cheap talk. Under mild assumptions we prove that equilibria have an interval-partition structure, where types of the Sender belonging to the same interval either all induce the same action through cheap talk or reveal their types through hard evidence. We also show that the availability of costly hard signals may reverse one of the important implications of the classical cheap talk model, namely, that diverging preferences always lead to less communication.

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1 Introduction

In his work on persuasion, Aristotle (347 B.C.E.) argues that a trained rhetor can rely on three fundamental tools in order to convince his audience: *Logos* (evidence and deduction), *Pathos* (listener emotions), and *Ethos* (speaker credibility). This classification seems appropriate in many economic interactions involving communication. For example, an expert advising a decision maker on policy can use hard data, or make unverifiable statements, or simply appeal to the decision maker's emotions. A fundamental question of rhetoric is: What mixture of these three tools of persuasion is most effective in different situations?

In game theoretical models of communication (sender-receiver games) *Logos* corresponds to verifiable, hard information, *Ethos* to unverifiable pure cheap talk, and *Pathos* remains to be quantified.[1] In this paper, we investigate certain tradeoffs between hard and soft information transmission (*Logos* and *Ethos*) in a pure communication game à la Crawford and Sobel (1982). Our most interesting results concern the change in the use and optimal mixture of hard and soft signals, and the amount of information transmitted in equilibrium, as the interests of the expert and the decision maker become less aligned.

We consider a model of information transmission where the costly provision of type-revealing 'hard evidence' is available in addition to conventional cheap talk. Under certain conditions we prove that equilibria exist and have an interval-partition structure, where types of the Sender belonging to the same interval either all induce the same action through cheap talk or reveal their types through hard evidence. We also show that the availability of costly hard signals can reverse one of the most robust implications of the classical cheap talk model, namely, that diverging preferences always lead to less communication.

Our model and results are relevant in a number of applications. By characterizing equilibria in which both cheap talk messages and costly hard signals are used, we show how formal evaluations and informal assessments can co-exist in education, management control, and firms' internal communication systems. We argue that

[1] A hard signal sent by the sender is evidence that a particular state of nature has obtained, and so can be used by the receiver as a basis for logical inference (*Logos*). The meaning of a cheap talk message in equilibrium depends on the set of sender types who send that message, and therefore can be interpreted as deriving from the reputation or credibility of the sender (*Ethos*). We are unaware of communication models where the players' emotions (*Pathos*) are explicitly taken into account.

when evidence production is costly, it is the most extreme types of the sender who provide hard signals in order to avoid being pooled with moderate types. Most interestingly, in certain cases an *increase in the conflict* between the expert's and the decision maker's preferences leads to a heavier reliance on hard evidence. As a result, *more information* is transmitted in equilibrium and the decision maker is better off.

To understand the intuition behind the latter result, consider the leading, uniform-quadratic example of Crawford and Sobel (1982). That is, assume that the Sender's privately known type is uniformly distributed on $[0, 1]$, the Receiver wants to set her action equal to the Sender's type whereas the Sender has a constant, positive bias; both parties have quadratic loss functions. In the babbling equilibrium, which always exists and in which the Receiver learns nothing about the Sender's type, the Receiver's action is $\frac{1}{2}$. Because of the constant, positive Sender bias, it is the highest type of the Sender that would gain the most by revealing his type. If a type-revealing signal were available at no cost then this type would use it, and the well known process of 'unraveling' would begin, leading to all types sending hard signals. However, if the hard signal is costly, but not too costly, then as the interval of types that babble shrinks due to partial unraveling, the utility gain of the highest babbling type from switching to the hard signal decreases and eventually equals the cost of the hard signal. In equilibrium, there is a marginal type that is indifferent between being pooled with the interval of types that babble and revealing his type. Now suppose that the Sender's bias becomes larger. Because the Receiver's action is targeting the *average* of the types that babble (which is strictly below the marginal type) and the loss function is strictly convex, the utility of the marginal type falls by more if he babbles than in case he reveals his type. Therefore this type strictly prefers to send the hard signal. The set of types sending a hard signal increases and the interval of those that babble decreases. In the new equilibrium more information is transmitted to the Receiver.

An area where cheap talk games have been applied particularly successfully is the study of parliamentary institutions. In many political systems legislation is proposed by committees that are better informed about policy consequences than the median voter in the legislature. The leading theoretical framework in which this legislative process is formulated, Gilligan and Krehbiel (1987), is partly based on Crawford and Sobel's (1982) sender-receiver game with a uniform type distribution, constant bias and quadratic utilities.[2] The standard model only allows for the transmission of 'soft' information in the form of a proposal from the committee to the floor. But in reality committees can also produce reports, transcripts of hearings and expert testimony in addition to a proposed bill. The provision of this type of hard evidence is clearly costly for the committee; for example they need resources to summon and interview witnesses. This is exactly the feature that our model attempts to capture. Contrary to received wisdom, our results indicate that a committee whose median member is more biased relative to the median legislator may provide more information to the legislature in the form of hard evidence, which leads to better decisions from the perspective of the median legislator.

Our model considers free and unverifiable as well as costly and verifiable signals in a communication setting. The canonical model of the transmission of unverifiable information is that of Crawford and Sobel (1982) with its well-known prediction regarding partition (imperfectly separating) equilibria. In a similar setup, Grossman (1981) and Milgrom (1981) show that when information is verifiable, the unique equilibrium involves perfect separation of all sender types. Their unraveling argument applies when the sender can send hard signals at no cost. Seidmann and Winter (1997) extend the result for significantly more general environments, assuming partial provability. Mathis (2008), in turn, generalizes the results of Seidmann and Winter (1997), and derives necessary and sufficient conditions for a fully revealing equilibrium to exist under partial provability.

Verrecchia (1983) points out that if disclosure of hard information is *costly* then full revelation may fail to occur. In his model a firm's management can either verifiably disclose the firm's profitability at a certain cost or refuse to do so. The market has rational expectations and values the firm's shares according to the disclosure (or the lack thereof). In equilibrium the firm discloses its profitability when it is above a certain positive threshold and withholds the information otherwise. At the threshold the firm is indifferent between being valued fairly while incurring the disclosure cost, and being perceived as an average below-threshold firm while saving the cost of disclosure. In Verrecchia (1983) the underlying communication model is a monotonic signaling game (the firm always wants to be perceived having as high profits

[2]Specifically, Gilligan and Krehbiel's (1987) model features four "legislative games." Of these, the one in which the committee is informed and the floor's policy choice is unrestricted is equivalent to Crawford and Sobel's (1982) model in the uniform-quadratic case with a constant bias.

as possible), while in ours it is a Crawford-Sobel (non-monotonic) signaling game.[3] Therefore, his model can be seen as a special case of ours when the sender's bias is infinitely large and all but one of the free, unverifiable messages are disallowed. Naturally, in his model, the question whether a greater bias leads to more or less information transmission does not arise.

Other models with hard information transmission restrict the sender's (or senders') ability to prove what the state of nature is, and get only partial revelation. Lipman and Seppi (1995) provide conditions on the sets of available messages under which it is feasible for one or more senders to identify the state.[4] In Shin (1994) senders with opposing interests have hard evidence that the state lies above or below certain privately-known thresholds. In equilibrium, however, the senders suppress all information that is unfavorable for their own bias, and, interestingly, a more informed sender bears more of the burden of proof.[5] Another model of communication with partial verifiability is that of Glazer and Rubinstein (2004); Rubinstein and Glazer (2006). They study an environment where the sender can provide a hard signal about only one of the two dimensions of the state of nature. The receiver can commit to a binary action—accept or reject—which she would like to condition on the state. In contrast, the sender is commonly known to prefer one of the actions in all states.[6] Glazer and Rubinstein characterize equilibria and receiver-optimal communication mechanisms.

Our model is different from the above models of partial verifiability in that we expand the canonical model of cheap talk of Crawford and Sobel (1982) so that there is a direct (payoff) cost of sending a hard signal without altering other key aspects of the model, such as the state-dependence of the sender's preferences and the continuous nature of receiver's action. A related strand of literature (see Ottaviani

[3]Another difference is that Verrecchia (1983) does not allow free, unverifiable (cheap-talk) messages while we do, and therefore there is only one cheap talk induced action in equilibrium.

[4]The issue of implementation (mechanism design) when the agent(s) can partially prove the state is studied by Deneckere and Severinov (2008) and papers cited therein (going back to Green and Laffont (1986)). Our paper is only marginally related to this literature as we study communication equilibria, not mechanisms.

[5]In a recent paper Dziuda (2011) uses a structure similar to Shin's in a single-expert communication game. In her model not all unfavorable hard information gets suppressed, but full revelation is still impossible.

[6]Note that as compared to the standard Crawford-Sobel cheap talk game, the state space here is more general (multidimensional), but the Receiver's action and the Sender's preferences are very much restricted (binary action, state-independent Sender preferences).

and Squintani (2006), Kartik et al (2007), and Kartik (2009)) introduces *costly lying* (or a credulous receiver, implying the same) in the same setting. In these models the sender announces his type, which is an unverifiable yet costly statement if he deviates from telling the truth. In contrast, we assume that it is costly to report the truth in a verifiable way. The structure of equilibria under costly lying is quite different from what we find under costly verification. For example, when lying is costly (with bounded state spaces and positive sender bias), low sender types separate by telling the truth and high types pool on the same lie. In contrast, in our model with comparable preferences but costly verification the high types separate by proving the state and the low types pool by sending free, unverifiable messages.

In the original Crawford-Sobel cheap talk model and most of its variants, better-aligned preferences lead to more communication and higher payoffs for the receiver. In an interesting recent paper Che and Kartik (2009) suggest a reason why and how this result may be overturned. They argue that getting advice (in the form of partially verifiable signals) from a more biased expert may be beneficial for the decision maker because a biased sender may have a greater incentive to acquire such signals in order to persuade the receiver. This result is obtained under the assumption that the Sender and the Receiver have different opinions, i.e., that they do not share the same prior about the state of nature. Our model is set up and works differently from theirs. Instead of studying the costly *acquisition* of partially verifiable signals, we model the Sender's incentives for costly (and full) *verification* of already existing private information when cheap talk messages are also allowed. Perhaps the most important difference, however, is that in their model a difference of opinion (different priors) plays an important role, whereas we assume common priors.

Though it does not feature hard information, Austen-Smith and Banks (2000) with its model of burning money is related to our work. The structure of an equilibrium with burned money is similar to the structure of our equilibrium involving hard information. However, there are significant differences. Agents who burn money send costly and unverifiable (cheap-talk) messages, and they signal their type with the cost they incur. Our Sender types can send hard information at a fixed cost. In the model of Austen-Smith and Banks fully revealing equilibria always exist, whereas in our model that is possible only if the cost of hard information is zero.

The paper is structured as follows. In Section 2 we set up a model of communication via cheap talk and costly hard (verifiable) signals. In Section 3 we prove that an interval-partition equilibrium exists, and analyze its properties. In Section 4 we show that in the uniform-quadratic case (which is frequently used in applications) our model implies that an increase in the sender's bias leads to more information transmission and a higher expected utility for the receiver. An extension (example) with a multidimensional state space is studied in Section 5. Section 6 concludes; omitted proofs are collected in an Appendix.

2 The model

There are two players, the Sender (he) and the Receiver (she). The Sender has private information about the payoff-relevant state of nature, which we represent by the realization of a one-dimensional random variable θ, and call his *type*. We assume that θ is distributed according to a continuous and positive density (pdf) f on a compact support normalized to $[0, 1]$. The Sender, having observed θ, sends a message m from a set $M \cup \{h_\theta\}$. Messages in M (a non-empty set) are non-verifiable (soft) signals because any type of the Sender can send them. The message h_θ is a verifiable (hard) signal because it can only be sent by type θ, thus it identifies the Sender.[7] The Receiver observes m and picks an action, $y \in \mathbb{R}$.

We assume that each player's payoff is strictly concave in y and that for any given Sender type, each player has a most-preferred action, which is finite. We denote the Sender's ideal point by $y^S(\theta)$, and the Receiver's by $y^R(\theta)$; call $b(\theta) \equiv y^S(\theta) - y^R(\theta)$ the Sender's bias. Following Crawford and Sobel (1982), we also assume that y^S and y^R are both strictly increasing and continuously differentiable, and that the sign of the Sender's bias is the same for all θ. Without loss of generality, we normalize $b(\theta) > 0$ for all θ. In the classical cheap talk model without hard signals, these assumptions guarantee that in any equilibrium, only a finite number of different actions are induced, and that the set of types inducing any given action form an interval.[8]

[7]We could allow hard signals of the form $\{h_T\}_{T \subseteq [0,1]}$ that can only be sent by types $\theta \in T$. A model with this richer structure is not required for obtaining our main results.

[8]See Crawford and Sobel (1982). The key assumption that guarantees interval-partition equilibria is that the parties' ideal points are monotonic. The partition is finite because the sign of the bias is constant. Gordon (2010) shows that if the Sender's bias switches signs over the type space then

We make three assumptions that differentiate our game of information transmission from that of Crawford and Sobel (1982). Our first assumption is that Sender types are able to send a type-revealing *hard signal*, and the cost of sending that signal is the same (and positive) for every type of the Sender. The second assumption concerns the Sender's gross utility (i.e., his payoff ignoring the cost of the hard signal): We assume that it only depends on the distance between the Sender's ideal point and the Receiver's action. That is, the Sender's loss from the mismatch between his ideal point and the Receiver's action is independent of his type (the state of nature), just as the cost of sending the hard signal is, which makes these two sources of disutility more easily comparable. Finally, we assume that the Sender's bias, i.e., the difference between the Sender's and the Receiver's ideal points, is weakly increasing in the Sender's type.

For clarity, we formally state our specific assumptions as follows.

Assumption 1: The Sender incurs a positive cost, c, for sending the hard (type-revealing) signal, h_θ, and this cost is independent of his type.

Assumption 2: The Sender's payoff depends on his type and the Receiver's action y only via the *distance* between y and $y^S(\theta)$.

By Assumptions 1 and 2, the payoff of the Sender with type θ from Receiver action y can be written as $U^S(y - y^S(\theta)) - \mathbf{1}_{m=h_\theta}c$, where U^S is symmetric about and maximized at zero (also strictly concave by an assumption made earlier), and $\mathbf{1}_X$ is the indicator function (which equals 1 if X is true and 0 otherwise).

Assumption 3: The Sender's bias, $b(\theta) \equiv y^S(\theta) - y^R(\theta)$ which is normalized to be positive, is *weakly increasing* in θ.

The role of this last assumption is to guarantee that in any equilibrium of the model with costly hard signals, Sender types that induce a given action through cheap-talk form an interval.[9] This result is established in Proposition 2. The following counterexample illustrates that when costly hard signals are available and $b(\cdot)$ is strictly decreasing, the set of types that induce the same action by sending the same cheap talk message need not be connected. Example 1 is ruled out by Assumption 3.

the number of equilibrium messages may be infinite.

[9]Assumption 3 is sufficient but not necessary for this result. We use this instead of weaker conditions on the concavity and single-crossing properties of the Sender's utility for expositional purposes.

Example 1. Assume that the Sender's type follows a symmetric, continuous distribution such that almost all of the mass is near $\theta = 0.5$, that is, for $\varepsilon > 0$ small, $\Pr(0.5 - \varepsilon < \theta < 0.5 + \varepsilon) > 1 - \varepsilon$. Let the Receiver's ideal point be $y^R(\theta) = \theta$, whereas the Sender's ideal point a smooth approximation of

$$y^S(\theta) = \begin{cases} 0.25 & \text{for } \theta \in [0, 0.2], \\ \theta + 0.05 & \text{for } \theta \in [0.2, 1]. \end{cases}$$

Both parties have quadratic loss functions and the cost of the hard signal is $c = 0.05$. Direct calculations reveal that if a cheap-talk message induces $y = 0.5$ then the net (after-cost) gain of the Sender from sending the hard signal is negative for $\theta \in [0, 0.1]$ as well as for $\theta \in [0.4, 0.6]$, but positive for θ near 0.2 as well as for all $\theta > 0.7$. For ε sufficiently small, the expected value of θ conditional on not falling into the ranges where the gain from sending the hard signal is positive is arbitrarily close to 0.5. Therefore, for ε sufficiently small, there is an equilibrium where the set of types that babble contains an interval $[0, \theta_1]$ with $\theta_1 < 0.2$, and another, disjoint interval in the neighborhood of 0.5; however some types in between send the hard signal. $\square$

The solution concept used in this paper is *perfect Bayesian equilibrium* (henceforth equilibrium, for short). Though in the Crawford-Sobel cheap talk game the notion of perfection does not add to the Bayesian-Nash equilibrium concept, it matters when hard signals are available.[10] In a Bayesian Nash equilibrium the Receiver can use incredible threats to force (some types of) the Sender to use the hard signal, but also, the Receiver can ignore hard signals when sent off the equilibrium path. Following the strategy of several studies going back at least to Milgrom (1981), we exclude such equilibria by requiring perfection.

It is well known that cheap-talk games usually exhibit a multiplicity of equilibria. There always exist multiple babbling equilibria, in which messages may be sent but do not affect the Receiver's action. Even equilibria that feature meaningful (if partial) communication can be reproduced by permuting messages. Thus the only truly meaningful feature of any equilibrium is the induced equilibrium mapping from types of the Sender to actions of the Receiver, and this mapping is what we focus on when describing our results.

[10]We thank an Associate Editor for bringing this matter to our attention.

3 Existence and properties of equilibria

The following Lemma is the key in proving both the existence and the interval-structure of equilibria in our model. It concerns the properties of the Sender's gross gain (as a function of his type) from sending the hard signal instead of inducing a given action through cheap talk. We show that this gross gain is strictly decreasing on an interval of low types, negative for types around the "average" type inducing the particular action via cheap talk, and positive for sufficiently high types.

Lemma 1. Fix $\underline{\theta}, \bar{\theta} \in [0,1]$ with $\underline{\theta} < \bar{\theta}$. Let $\theta_1 \in (\underline{\theta}, \bar{\theta})$ and $y_1 = y^R(\theta_1)$. There exists $\theta_0 \in [\underline{\theta}, \theta_1)$ such that $U^S(b(\theta)) - U^S(y^S(\theta) - y_1)$ is

(i) strictly decreasing in θ for all $\theta \in [\underline{\theta}, \theta_0)$,

(ii) negative for all $\theta \in (\theta_0, \theta_1)$,

(iii) strictly increasing in θ for all $\theta \in (\theta_1, \bar{\theta}]$.

Proof. See the Appendix. □

The shape of $U^S(b(\theta)) - U^S(y^S(\theta) - y_1)$ is illustrated in Figure 1.

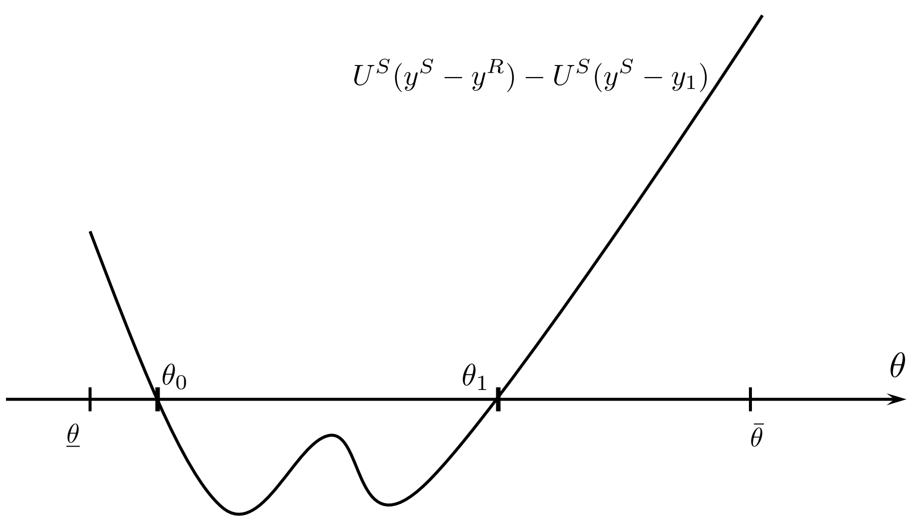

Figure 1: The gross gain from sending the hard signal

An immediate implication of Lemma 1 is that if there are any Sender types who prefer sending the hard signal at cost c compared to inducing $y_1 = y^R(\theta_1)$ through cheap talk, then they are located at the extreme(s) of an interval containing θ_1. This is so because the net gain from sending the hard signal instead of inducing y_1 via cheap talk is $U^S(b(\theta)) - U^S(y^S(\theta) - y_1) - c$, which is decreasing for low θ's, increasing for high θ's and negative in the middle (for θ's near θ_1). To put it differently:

Corollary 1. Given any interval $[\underline{\theta}, \bar{\theta}] \subseteq [0,1]$ and action $y_1 = y^R(\theta_1)$ with $\theta_1 \in (\underline{\theta}, \bar{\theta})$, the Sender types in $[\underline{\theta}, \bar{\theta}]$ that weakly prefer action y_1 over sending the hard signal at cost c form a nonempty, closed interval around θ_1.

For any interval $[\underline{\theta}, \bar{\theta}] \subseteq [0,1]$, let $\mu(\underline{\theta}, \bar{\theta})$ denote the Receiver's optimal action conditional on believing that the Sender's type belongs to the interval $[\underline{\theta}, \bar{\theta}]$.

Let $\underline{b} = \min_{\theta \in [0,1]} b(\theta)$, the minimum bias across all Sender types. (Under our assumptions all extrema discussed below are attained.) By Crawford and Sobel (1982), every cheap-talk equilibrium has an interval partition structure, with the length of each interval bounded below by $\underline{b}$. Therefore, if our game has an equilibrium in which the hard signal is not sent by any type, then the interval of types that induce the same Receiver action via cheap talk as type $\theta = 1$ does is at least $\underline{b}$ long. Having a positive bias, type $\theta = 1$ clearly prefers action $y^R(1)$ to $\mu(1 - \underline{b}, 1) < y^R(1)$. Let $\underline{c}$ be the cost that makes type $\theta = 1$ indifferent between revealing his type and the Receiver's response to learning $\theta \in [1 - \underline{b}, 1]$, that is,

$$\underline{c} = U^S(b(1)) - U^S(y^S(1) - \mu(1 - \underline{b}, 1)). \tag{1}$$

If $c < \underline{c}$ then in any equilibrium, type $\theta = 1$ sends the hard signal. Lastly, define

$$\bar{c} \;=\; \max_{\theta \in [0,1]} \left\{ U^S(b(\theta)) - U^S(y^S(\theta) - \mu(0,1)) \right\}. \tag{2}$$

This is the maximum utility gain any type of the Sender can obtain by sending the hard signal in the babbling equilibrium. Clearly, if $c > \bar{c}$ then there is an equilibrium (babbling) such that no type sends the hard signal. Since $\mu(0,1) \leq \mu(1 - \underline{b}, 1) < y^S(1)$, we have $U^S(y^S(1) - \mu(0,1)) \leq U^S(y^S(1) - \mu(1 - \underline{b}, 1))$. Thus the maximand in (2) at $\theta = 1$ weakly exceeds the right-hand side of (1), and therefore $\bar{c} \geq \underline{c}$.

Proposition 1. If $c < \bar{c}$ then there exists an equilibrium where some types of the Sender send the hard signal. If $c < \underline{c}$, all equilibria involve hard information.

Proof. For $\theta \in [0,1]$, let $S(\theta) \subseteq [0,1]$ denote the set of types that weakly prefer the outcome $y^R(\theta)$ to sending the hard signal at cost $c > 0$. Since the Sender with type θ (and types close by) strictly prefer the former, $S(\theta)$ is not empty. Indeed, it follows from Lemma 1 (see Remark 1) that $S(\theta)$ is an interval $[\underline{s}(\theta), \overline{s}(\theta)]$ such that $\underline{s}(\theta) \leq \theta \leq \overline{s}(\theta)$, with strict inequalities if $0 < \theta < 1$.

Define the function $g : [0,1] \to [0,1]$ by

$$g(\theta) = \left(y^R\right)^{-1} \left(\mu(\underline{s}(\theta), \overline{s}(\theta))\right). \tag{3}$$

All functions involved in the definition of g are continuous by assumption (by the continuity of the players' ideal points and the positive, continuous density of θ), moreover y^R is strictly increasing hence it is invertible. Therefore g is well-defined and continuous. By Brouwer's Fixed Point Theorem g has a fixed point θ^*. This determines an equilibrium where Sender types $\theta \in [\underline{s}(\theta^*), \overline{s}(\theta^*)]$ babble and induce $\mu(\underline{s}(\theta^*), \overline{s}(\theta^*)) = y^R(\theta^*)$, whereas the rest of the types send the hard signal.

Since $c < \bar{c}$, at least one type would strictly prefer sending a hard signal at cost c compared to the outcome of the babbling equilibrium, therefore hard information is indeed used in the equilibrium.

The second part of the Proposition has been established in the discussion leading to the definition of $\underline{c}$ in equation (1). □

We described the intuition behind the result in Proposition 1 in the Introduction. To recapitulate, if the Sender can costlessly reveal his type then the babbling equilibrium unravels. In contrast, if the cost is too high then no type of the Sender uses hard information. When the cost of hard information is strictly positive but not too large, the babbling equilibrium unravels only partially, and an equilibrium exists in which one action is induced through cheap talk, and other types of the Sender use hard information. The proof establishes the existence of one particular equilibrium in which hard signals are sent; it does not imply uniqueness nor that all equilibria must have a similar structure.

Our next result is that all equilibria of the game exhibit a certain *interval-partition property* familiar in cheap-talk games. More precisely, we show that the set of types that induce a given action via cheap talk is connected, whereas the set of types that send the hard signal is a finite union of intervals.

Proposition 2. In any equilibrium, there exist $a_0 = 0 \leq a_1 \leq \ldots \leq a_N \leq a_{N+1} = 1$ cutoff points such that in every interval (a_k, a_{k+1}), either all types in the interval induce the same action through cheap talk, or all send hard signals. Cheap-talk messages sent by types belonging to different intervals induce different actions.

Proof. Recall that $\underline{b} = \min_{\theta \in [0,1]} b(\theta)$ is positive. Consider two actions $y < y'$ induced by cheap-talk messages in an equilibrium. For types θ that choose y, action $y^R(\theta) + b(\theta)$ is closer to y than it is to y'. For types θ' that choose y', $y^R(\theta') + b(\theta')$ is closer to y' than to y. Since $y^S = y^R(\theta) + b(\theta)$ is continuous and increasing, there must be a type m such that $y^R(m) + b(m)$ is half-way between y and y'. Moreover, all types below m must prefer y to y', and vice versa for types above m. Since the Receiver's response to an interval ending at m is y, it must be that $y \leq y^R(m)$. But $y^R(m) + b(m)$ is half-way between y and y', and $b(m) \geq \underline{b}$, so $y' - y \geq \underline{b}$. Thus there can only be finitely many cheap-talk-induced actions in any equilibrium.

The preceding argument also establishes that there cannot be three types $\theta < \theta' < \theta''$ such that θ and θ'' induce the same action through cheap talk, while θ' induces another action through cheap talk. By Lemma 1, no hard signal senders can be wedged in between types inducing the same, given action via cheap-talk messages either. Therefore, types inducing any given action through cheap talk must form an interval, and there are finitely many such intervals. □

If the bias $b(\cdot)$ is very large for all types then there is no equilibrium in which cheap-talk messages induce more than one action (i.e., all cheap-talk messages are equivalent to babbling). But this means that if the cost c is not too high then an interval of types including $\theta = 1$ prefer to send the hard signal. Thus for large biases and relatively small cost c, the unique equilibrium outcome involves one action induced via cheap talk by types below a cutoff type $\hat{\theta}$, and the hard signal being sent by types above $\hat{\theta}$.

The way equilibrium partitions are structured depends on the environment. In general, all we can say is that types sending the costly hard signal are located either at the extremes of the type space or in between types inducing different actions through cheap talk. The following Proposition establishes conditions under which the types that send the hard signal form a *single interval* near the upper boundary of the type space.

Proposition 3. Assume that the cumulative distribution function (cdf) of θ is concave and the Receiver's payoff is $U^R(y,\theta) = -(y-\theta)^2$. In every equilibrium where some Sender type(s) send the hard signal, all such types belong to the partition element that includes $\theta = 1$.

Proof. See the Appendix. □

The structure of equilibrium with hard signals under the hypotheses of Proposition 3 is illustrated in Figure 2. There are two actions induced by cheap-talk messages in this equilibrium, whereas types in $(a_2, 1]$ send hard signals, that is, identify themselves at cost c.

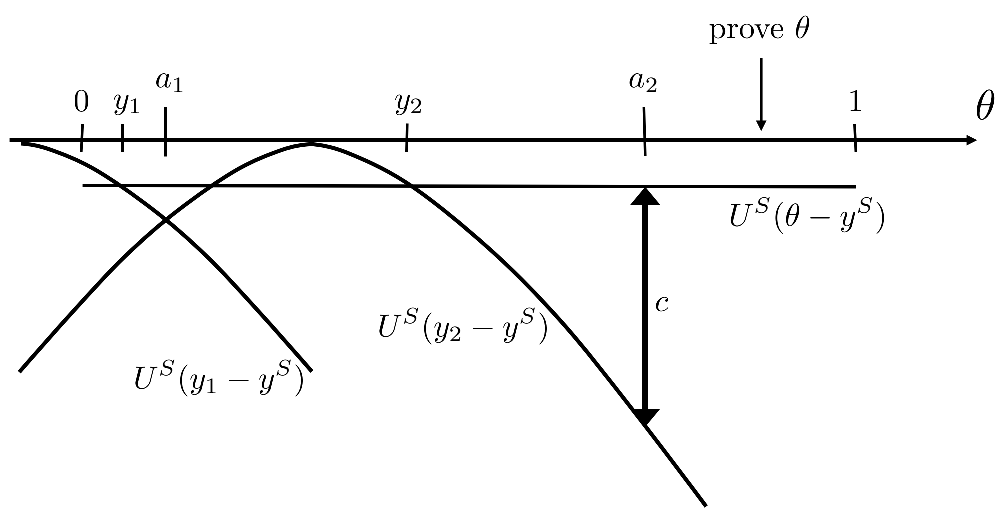

Figure 2: The structure of equilibrium under the hypotheses of Proposition 3.

A natural question to ask is whether the interval partition underlying the equilibrium mapping from Sender types to Receiver actions is unique, at least for a given number of partition elements. In the corresponding information-transmission model without costly hard signals Crawford and Sobel (1982) proposed Condition M for the purpose of verifying the uniqueness of the equilibrium partition. Let $a_0 = 0$ and set $a_1 \in (0,1)$, then for $i = 1,\ldots,k-1$ compute a_{i+1} recursively, as long as $a_{i+1} \leq 1$, so that the Sender with type a_i is indifferent between $\mu(a_{i-1}, a_i)$ and $\mu(a_i, a_{i+1})$. (Given a_1 all subsequent thresholds are uniquely determined by the indifference conditions.)

Condition M holds if for all $i = 2, \ldots, k$, a_i is strictly increasing in a_1. It turns out that even the conditions used in Proposition 3 (which are, in addition to Assumptions 1-3, that the Receiver's ideal point is θ, distributed according to a concave cdf) coupled with quadratic loss functions are not sufficient to guarantee Condition M and the uniqueness of the equilibrium partition (in our model or in Crawford and Sobel's). This is illustrated by the following example.

Example 2. Let $U^R(y, \theta) = -(y - \theta)^2$ and $b(\theta) = 0.01$; assume the cost of hard information is $c = 0.01$. Let θ be distributed according to the probability density

$$f(\theta, \varepsilon) = \begin{cases} 5.5 & \text{if } \theta < 0.1 - \varepsilon \\ 3 - \frac{5}{2\varepsilon}(\theta - 0.1) & \text{if } 0.1 - \varepsilon \leq \theta \leq 0.1 + \varepsilon \\ 0.5 & \text{if } \theta > 0.1 + \varepsilon, \end{cases}$$

where $\varepsilon > 0$. The density is continuous and weakly decreasing, so the cdf of θ is continuously differentiable and concave. As $\varepsilon \to 0$, $f(\theta, \varepsilon)$ converges to a density that is a discontinous step function with $f(\theta) = 5.5$ for $\theta < 0.1$ and $f(\theta) = 0.5$ for $\theta > 0.1$. We use the limiting density for the calculation of expected values, which are arbitrarily precise for ε sufficiently small. Condition M does not hold: Starting with $a_1 = 0.09$ we get $a_2 \approx 0.28$, whereas for $a'_1 = 0.1$ we have $a'_2 \approx 0.24$ by the Sender's indifference condition. Direct calculations reveal that the failure of Condition M leads to a multiplicity of equilibria featuring cheap talk messages as well as hard information transmission. For example, there are three distinct equilibrium partitions featuring two actions induced by cheap talk and other types sending hard signals.[11] $\square$

The equilibrium partition is unique in the specification of our model with a uniform type-distribution, type-invariant bias, and quadratic loss functions. This specification (without the availability of costly hard signals) is widely used in applications, for example, in political theory (see Gilligan and Krehbiel (1987)). When costly hard signals are available, this model has other interesting properties as well. This is what we explore in the next section.

[11] Details of these calculations can be found in an online appendix at www.adamgalambos.com under Research.

4 Disagreement and evidence production

An intuitive and often cited feature of communication games is that if the parties' preferences diverge, less information is transmitted. The most interesting consequence of the availability of costly hard signals in our model is that a more severe disagreement between the Sender and the Receiver can yield *more* communication between the parties, and higher welfare for the Receiver. Of course, this phenomenon does not arise in every environment, but it can be observed in the leading specification of Crawford-Sobel cheap talk models, the uniform-quadratic case with a constant bias. In Proposition 4 we claim that when the state is distributed uniformly on $[0, 1]$, the Receiver's ideal point is $y^R(\theta) = \theta$, the Sender's is $y^S(\theta) = \theta + b$, and with quadratic loss functions representing the parties' preferences, a small increase in b *almost always* leads to an increase in information transmission and Receiver's welfare.

Before we state and prove the proposition it may be useful to discuss what "increased information transmission" exactly means and how it can be the result of a larger bias. Since the uniform-quadratic specification satisfies the assumptions of Proposition 3, in an equilibrium with $(N + 1)$ positive-length partition elements involving hard signals, the types that send the hard signal belong to the interval $[a_N, 1]$, as seen in Figure 2. The Sender with type a_N is indifferent between (i) revealing his type at cost c and incurring a loss b^2, and (ii) sending the soft message corresponding to interval $[a_{N-1}, a_N]$ for free and incurring a loss of $(b+(a_N - a_{N-1})/2)^2$. As the bias increases, the increase in b^2 is smaller than the increase in $(b + (a_N - a_{N-1})/2)^2$ due to the strict convexity of the quadratic loss function. Therefore, *if the equilibrium partition remained the same* after b becomes larger, type a_N would strictly prefer to send the hard signal and the interval of types sending the hard signal would expand.

However, as b increases, the cutoffs $a_1, \ldots, a_{N-1}$ determining the equilibrium partition do not remain the same. What the proof of Proposition 4 establishes is that as the bias increases, all cutoffs decrease, moreover, the distances between adjacent cutoffs decrease as well. Therefore, each interval of types sending a given soft message shrinks, while the interval of types sending the hard signal expands. Though this does not lead to a finer equilibrium partition (the two are incomparable), it represents a clear increase in information transmission as measured in terms of entropy or the Receiver's payoff.

Proposition 4. Assume that θ is uniform on $[0, 1]$; $y^R(\theta) = \theta$, $y^S(\theta) = \theta + b$ with $b > 0$, and both players have quadratic loss functions. Fix an equilibrium in which hard signals are sent by some Sender types. An infinitesimal increase in b almost always increases the amount of information transmitted in equilibrium and the Receiver's expected payoff. Specifically, the length of every interval of types using cheap talk decreases, and the interval of types sending hard signals increases.

Proof. See the Appendix. □

Note that if the bias increases substantially then the number of actions induced through cheap talk messages in equilibrium may decrease. However, this is not the case for infinitesimal changes in b when the initial partition is non-degenerate (each interval has positive length). This is why the Proposition claims that a small increase in b "almost always" increases information transmission. In the proof of the Proposition, we derive an explicit formula for the equilibrium partition as a function of the number of cheap talk messages. Thus the equilibrium is unique for a given the number of distinct actions induced by cheap talk. Our comparative statics exercise, comparing two equilibria with the same number of actions induced through cheap talk, therefore makes sense.

In Proposition 4, a small increase in b leads to a decrease in the length of each cheap-talk interval and an increase in the interval of types sending hard information. This makes it unambiguous that information transmission increases. The assumption that b is constant is important for this result. Without it, even under the other assumptions, an increase in the bias does not necessarily shorten the intervals of types using cheap talk messages, as the following example shows.

Example 3. Assume that θ is uniform on $[0, 1]$, $y^R(\theta) = \theta$, $y^S(\theta) = \theta + b(\theta)$, where $b(\theta) = \theta^2/5 + 0.0001$. Both players have quadratic loss functions, and the cost of a hard signal is $c = 0.05$. Direct calculations reveal that the equilibrium partition (with three actions induced through cheap talk) is $(0, 0.171154, 0.366144, 0.668782, 1)$. Now change the bias to $b(\theta) = \theta^2/5 + 0.0002$. Under this new, pointwise greater bias, the equilibrium partition is $(0, 0.170768, 0.365664, 0.668329, 1)$. Each cutoff point decreased, but the length of the third cheap-talk partition element *increased*.[12] □

[12]Details of these calculations are available in an online appendix at `www.adamgalambos.com` under Research.

Finally, we note that in the uniform-quadratic specification with constant bias (used in Proposition 4), a decrease in the cost of the hard signal would also increase the amount of information transmitted in equilibrium as well as the Receiver's expected payoff. Consider an equilibrium with some types sending hard information. The boundary type a_N, which is indifferent between inducing the action $\mu(a_{N-1}, a_N)$ through cheap talk or sending the hard signal, will strictly prefer the hard signal if c decreases. Thus a_N will decrease, and the interval of types sending the hard signal will expand. The proof of Proposition 4 shows that the length of every cheap talk interval in the equilibrium partition shortens if a_N decreases (see equation 11 in the Appendix). Since the Receiver's utility is the negative of the variance of θ conditional on the information revealed by the Sender, it increases as the cost of hard signal decreases.

5 Extension to multidimensional state spaces

If the Sender's information is multidimensional, information can be transmitted through cheap talk even if the preferences of the Sender and the Receiver are very different. In fact, if the Sender's bias is symmetric across the two states, then *comparative information* can always be transmitted in a cheap talk equilibrium no matter how divergent preferences are (Chakraborty and Harbaugh (2007)). In the comparative cheap talk equilibrium the Sender reports which coordinate of the state of nature is the largest, second largest, and so on, without revealing the actual values. But even in this setup, the availability of costly hard information can change equilibria and improve information transmission. We demonstrate through an example how hard information can be combined with cheap talk in a model with two-dimensional state space. Our main insights apply in this example. First, the Sender types that engage in hard information transmission are "extreme" (located near the boundary of the state space). Second, an increase in the Sender's bias increases the informativeness of the equilibrium. The example also exhibits an interesting feature that is qualitatively different from the one-dimensional case: a hard signal sent about one dimension can, at the same time, provide information about the other dimension.

Suppose the Sender's type (or state) is $\theta = (\theta_1, \theta_2) \in [0, 1]^2$, and the prior distribution is uniform. The Receiver's ideal point is (θ_1, θ_2), whereas the Sender's is $(\theta_1 + b, \theta_2 + b)$, where $b > 0$. Both have quadratic loss functions, that is, each player's utility is the negative of the square of the Euclidean distance between the action taken by the Receiver and the player's ideal point. This is a special case of the symmetric model of Chakraborty and Harbaugh (2007), and a generalization of Crawford and Sobel's uniform-quadratic example (also the basis of our model in the previous section) for a two-dimensional type space.

If there are no hard signals then, for any b, there exists a comparative cheap-talk equilibrium with two induced actions. The Sender reveals whether or not $\theta_1 \geq \theta_2$, and the Receiver replies with action $(2/3, 1/3)$ if $\theta_1 \geq \theta_2$ and action $(1/3, 2/3)$ otherwise. (The Receiver picks the expected value of the Sender's type conditional on falling into the half of the type space revealed by comparative cheap talk.) We analyze the effect of the availability of costly hard information on this equilibrium.

Suppose that at cost $c > 0$ the Sender can prove either of the coordinates of (θ_1, θ_2). (Contrast this with Glazer and Rubinstein's persuasion game where exactly one coordinate can be proven for free, there are no soft messages, and the Receiver's action is binary.) What is the maximum level of c where hard information may be sent in equilibrium? In the two-partition equilibrium, type $(1, 1)$ has the greatest loss, $(1/3 + b)^2 + (2/3 + b)^2$. If he reports that $\theta_1 \geq \theta_2$ and reveals either coordinate (which is 1), then he obtains a loss of $2b^2$. He is indifferent between the comparative cheap talk equilibrium and revealing costly hard information if

$$c = \left(\frac{2}{3} + b\right)^2 + \left(\frac{1}{3} + b\right)^2 - 2b^2 = \frac{5}{9} + 2b.$$

As long as $c < 5/9 + 2b$, some of the highest Sender types prefer sending the costly hard signal to the outcome of the comparative cheap talk equilibrium.

An equilibrium with hard information transmission will look like the one depicted in Figure 3. In this equilibrium the Sender announces whether or not $\theta_1 \geq \theta_2$; moreover, if the Sender's type falls into either of the shaded areas then he verifies the largest coordinate. For example, if the state is $x = (x_1, x_2)$ as shown in the figure, the Sender reveals that the first coordinate exceeds the second coordinate and verifies x_1. If the Sender does not verify a coordinate then the Receiver's reaction is either a or a' (depending on which coordinate is reported to be greater than the other), whereas if he does verify one of the coordinates (which is claimed to be greater than the other) then the Receiver chooses the midpoint of the interval that is consistent with his report. For example, when the Sender's type is either x or z as in Figure 3, the verified coordinate is $x_1 = z_1$ and the Receiver picks $y = (x_1, (x_1 + x_2)/2)$. In the rest of this section we argue that this is indeed an equilibrium when c is close to $5/9 + 2b$. The reason for assuming c near the upper bound is to ensure that the area where either coordinate is verified is small in the neighborhood of the highest type.

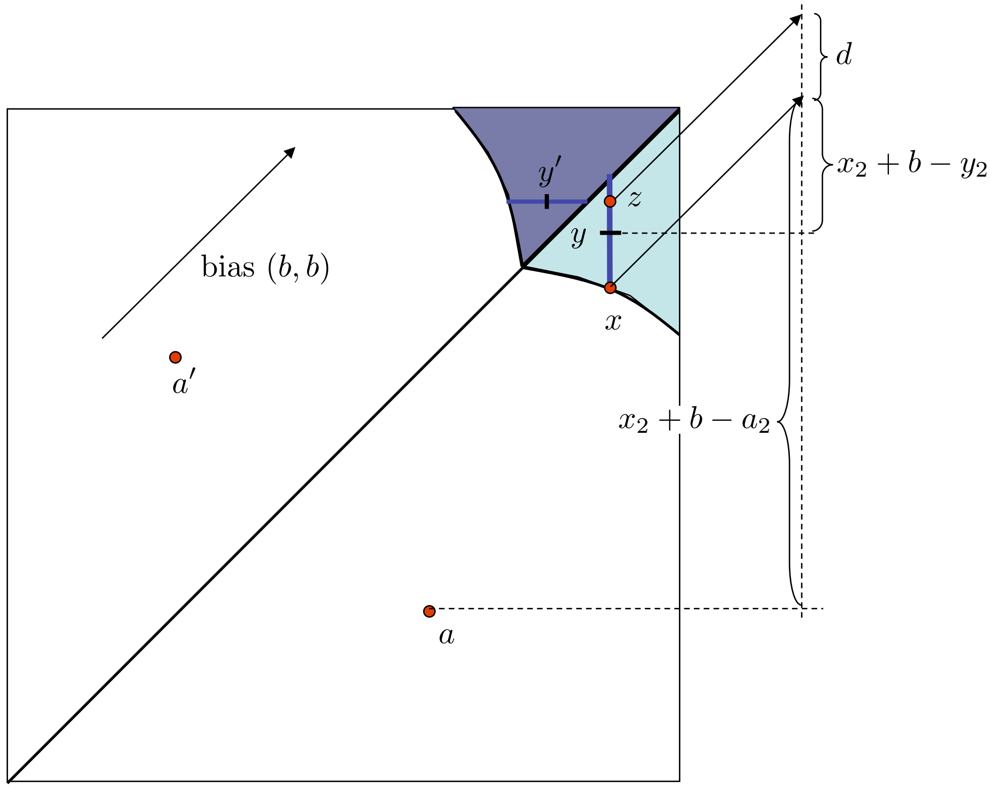

Figure 3: Types in the shaded areas send hard information

For a given pair of actions that can be induced through comparative cheap talk, type $x = (x_1, x_2)$ with $x_1 \geq x_2$ is indifferent between verifying x_1 and using comparative cheap talk that induces a if and only if

$$(x_1 + b - a_1)^2 + (x_2 + b - a_2)^2 = b^2 + \left(x_2 + b - \frac{x_1 + x_2}{2}\right)^2 + c, \qquad (4)$$

where the left-hand side is this Sender type's loss from inducing a through cheap talk and the right-hand side is his loss from verifying x_1.

We claim that if (4) holds then Sender type z such that $z_1 = x_1 \geq z_2 > x_2$ strictly prefers to verify the first coordinate, $z_1 = x_1$. To see this, note that $(z_1, z_2) = (x_1, x_2 + d)$ with $d > 0$. The Sender's loss from inducing a is

$$(z_1+b-a_1)^2+(z_2+b-a_2)^2 = (x_1+b-a_1)^2+(x_2+b-a_2)^2+2(x_2+b-a_2)d+d^2, \quad (5)$$

whereas his loss from verifying the first coordinate is

$$(z_2+b-y_2)^2+b^2+c = \left(x_2+b-\frac{x_1+x_2}{2}\right)^2+b^2+c+2\left(x_2+b-\frac{x_1+x_2}{2}\right)d+d^2. \quad (6)$$

By (4) and $d > 0$, we find that the loss from cheap talk, (5), exceeds that from verifying the first coordinate, (6), if and only if $a_2 < (x_1 + x_2)/2$. This inequality indeed holds provided that the boundary type x that is indifferent between inducing a through cheap talk and verifying x_1 at cost c exists, and c is sufficiently close to $5/9 + 2b$, because in this case x_1 and x_2 are both near 1 and a_2 cannot exceed 1/3, the second coordinate of the mean type conditional on $\theta_1 \geq \theta_2$.

Note that $a = (a_1, a_2)$ is determined endogenously as the expected state when $\theta_1 \geq \theta_2$ and θ_1 is not verified. For each a in the lower triangle, i.e. $0 \leq a_2 \leq a_1 \leq 1$, first compute all (x_1, x_2) such that (4) holds (these are the boundary types in the lower triangle that are indifferent between inducing a and verifying x_1), and let $S(a)$ be the set of all $0 \leq \theta_2 \leq \theta_1 \leq 1$ such that if there is a boundary type with $x_1 = \theta_1$ then $\theta_2 \leq x_2$. That is, $S(a)$ is the set of types in the lower triangle that weakly prefer inducing a rather than verifying θ_1 at cost c. $S(a)$ is non-empty (for c near the upper bound) and closed, and varies continuously with a. Thus $g(a) = \mathbb{E}[\theta | S(a)]$ is continuous and maps the lower triangle into itself. Therefore g has a fixed point, which is the equilibrium value of a. We conclude that a comparative cheap talk equilibrium with hard information transmission by some of the highest types indeed exists for c near $5/9 + 2b$.

In the equilibrium described above, the hard signal sent by a type $\theta = z$ (as in Figure 3) transmits more information than it directly proves. From the fact that the Sender verifies $z_1 = x_1$ the Receiver learns $z_2 \geq x_2$.

As the Sender's bias increases, the region in the type space where hard information is sent expands for the same reason as in the one-dimensional, uniform-quadratic model with constant bias. Types on the boundary between the hard-information and cheap-talk regions characterized by (4) strictly gain from verifying the highest coordinate when b infinitesimally increases. This is so because in (4) the left-hand side increases faster in b than the right-hand side does. There is more information transmitted in equilibrium when b is larger, and the Receiver is better off.

6 Conclusion

We have studied the equilibrium "mixture" of evidence-based, costly hard signals and conventional cheap talk in an information transmission game with bias. We have shown that the possibility of sending costly hard information in a Crawford–Sobel cheap talk game can be significant for the model's predictions. We have provided mild conditions under which all equilibria of the game are "interval-partition" equilibria. Moreover, if the Receiver's ideal point coincides with the Sender's type which follows a concave distribution, then hard information is sent by the highest Sender types.

Most interestingly, in the uniform-quadratic specification of the model with a constant Sender bias, if hard information is sent in an equilibrium, then an infinitesimal increase in the Sender's bias almost always leads to *more* information transmission and a higher expected payoff for the Receiver. Consequently, if hard information can be transmitted by the Sender at not too great a cost, then the Receiver might be better off choosing a more biased Sender. A legislature, for example, might prefer a more biased committee making a proposal if the committee can transmit moderately costly hard information such as expert testimony.

Much work remains to be done regarding the exact modeling of different forms of communication (*Logos*, *Ethos* and *Pathos* is just one classification), and very little is known about the tradeoffs that a speaker faces when choosing his arguments or even his audience.

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7 Appendix: Omitted Proofs

Proof of Lemma 1. If $y^S(\underline{\theta}) \geq (y_1 + y^R(\underline{\theta}))/2$ then let $\theta_0 = \underline{\theta}$. If $y^S(\underline{\theta}) < (y_1 + y^R(\underline{\theta}))/2$ then let $\theta_0 \in (\underline{\theta}, \theta_1)$ be the unique solution to $y^S(\theta_0) = (y_1 + y^R(\theta_0))/2$. Such a solution exists because $y^S(\theta)$ increases faster than $y^R(\theta)/2$ by the (weak) monotonicity of $b(\theta) \equiv y^S(\theta) - y^R(\theta)$, all involved functions are continuous, and $y^S(\theta_1) > y^R(\theta_1) = y_1$.

To see part (i), first note that for all $\theta \in [\underline{\theta}, \theta_0)$, $y^S(\theta) - y_1$ is negative and strictly increasing, and so $U^S(y^S(\theta) - y_1)$ whose argument is negative is strictly increasing as well. On the other hand, $U^S(b(\theta))$ is weakly decreasing in θ because $b(\theta)$ is positive and weakly increasing, and U^S is decreasing when its argument is positive. Therefore the difference $U^S(b(\theta)) - U^S(y^S(\theta) - y_1)$ is strictly decreasing in θ for $\theta \in [\underline{\theta}, \theta_0)$.

For all $\theta > \theta_0$, we have $y^S(\theta) > (y_1 + y^R(\theta))/2$ because the inequality holds weakly at $\theta = \theta_0$ by construction, and the left-hand side increases faster in θ than the right-hand side does. For all $\theta < \theta_1$, $y^R(\theta) < y_1$. Therefore, for all $\theta \in (\theta_0, \theta_1)$, the Sender's ideal point is closer to y_1 than it is to $y^R(\theta)$, hence $U^S(b(\theta)) < U^S(y^S(\theta) - y_1)$, and part (ii) follows.

Finally, we establish part (iii). The derivative of $U^S(b(\theta)) - U^S(y^S(\theta) - y_1)$ with respect to θ is

$$\left[\dot{U}^S(y^S(\theta) - y^R(\theta)) - \dot{U}^S(y^S(\theta) - y_1)\right] \dot{y}^S(\theta) - \dot{U}^S(y^S(\theta) - y^R(\theta))\dot{y}^R(\theta),$$

where we used $b(\theta) = y^S(\theta) - y^R(\theta)$ and let $\dot{}$ denote the derivative of the corresponding function. If $\theta > \theta_1$ then $y^R(\theta) > y_1$, hence $y^S(\theta) - y^R(\theta) < y^S(\theta) - y_1$. Therefore, by the concavity of U^S the bracketed expression is positive. By assumption, $\dot{y}^S(\theta)$ and $\dot{y}^R(\theta)$ are positive. However, $\dot{U}^S(y^S(\theta) - y^R(\theta)) < 0$ because $y^S(\theta) > y^R(\theta)$. Therefore the derivative of $U^S(b(\theta)) - U^S(y^S(\theta) - y_1)$ with respect to θ is positive for $\theta > \theta_1$. □

Proof of Proposition 3. Assume towards contradiction that an equilibrium includes a cheap-talk partition element $[a_i, a_{i+1}]$ preceded by a hard-information partition element. Then a_i is indifferent between sending the costly hard signal and a cheap talk message inducing $\mu(a_i, a_{i+1})$. That is,

$$U^S(b(a_i)) - c = U^S(\mu(a_i, a_{i+1}) - y^S(a_i)). \tag{7}$$

Recall $b(a_i) > 0$. To see that the argument of U^S is positive on the right-hand side as well, note that $y^S(a_i) = a_i + b(a_i)$ and $\mu(a_i, a_{i+1}) > a_i$ imply $y^S(a_i) - \mu(a_i, a_{i+1}) < b(a_i)$. Therefore, if $y^S(a_i) \geq \mu(a_i, a_{i+1})$ then $U^S(\mu(a_i, a_{i+1}) - y^S(a_i)) = U^S(y^S(a_i) - \mu(a_i, a_{i+1})) > U^S(b(a_i))$ because and U^S is symmetric about 0 and decreasing when its argument is non-negative. However, the latter inequality contradicts (7) and $c > 0$.

Because $U^R(y, \theta) = -(y - \theta)^2$ the Receiver's equilibrium response is $\mu(a_i, a_{i+1}) = E[\theta|\theta \in [a_i, a_{i+1}]]$. Since the cdf of θ is concave, we have $\mu(a_i, a_{i+1}) \leq (a_i + a_{i+1})/2$, and so $\mu(a_i, a_{i+1}) - a_i \leq a_{i+1} - \mu(a_i, a_{i+1})$. Therefore, by $b(a_i) > 0$,

$$\mu(a_i, a_{i+1}) - a_i - b(a_i) < a_{i+1} - \mu(a_i, a_{i+1}) + b(a_i).$$

We have shown that the left-hand side is positive, therefore both sides are. Since U^S is strictly decreasing when its argument is positive,

$$U^S(\mu(a_i, a_{i+1}) - a_i - b(a_i)) > U^S(a_{i+1} - \mu(a_i, a_{i+1}) + b(a_i)).$$

From this, equation (7), and $y^S(\theta) = \theta + b(\theta)$,

$$U^S(b(a_i)) - c > U^S(a_{i+1} - \mu(a_i, a_{i+1}) + b(a_i)).$$

Using $b(a_{i+1}) \geq b(a_i)$ and the fact that U^S is decreasing on positive domain,

$$U^S(b(a_{i+1})) - c > U^S(a_{i+1} - \mu(a_i, a_{i+1}) + b(a_{i+1})).$$

This means that Sender type a_{i+1} strictly prefers sending the hard signal to sending any cheap-talk message corresponding to $[a_i, a_{i+1}]$, which is a contradiction. □

Proof of Proposition 4. Denote the interval-endpoints of the equilibrium partition by $a_0 = 0 < a_1 < \cdots < a_N < a_{N+1} = 1$. As we showed in Proposition 3, for $k = 0, \ldots, N-1$, types in (a_k, a_{k+1}) induce the action $\mu(a_k, a_{k+1})$ through cheap talk, while $\theta \in (a_N, a_{N+1})$ produce hard evidence and induce action θ.

For all $k = 1, \ldots, N-1$, type a_k is indifferent between pooling with types in (a_{k-1}, a_k) and those in (a_k, a_{k+1}). Using $\mu(a_k, a_{k+1}) = (a_k + a_{k+1})/2$ and the quadratic loss function, the corresponding arbitrage condition is:

$$-\left(a_k + b - \frac{a_{k-1} + a_k}{2}\right)^2 = -\left(\frac{a_k + a_{k+1}}{2} - a_k - b\right)^2.$$

The expressions inside the parentheses are positive, so we take square roots to get the second-order linear difference equation $a_{k+1} = 2a_k - a_{k-1} + 4b$ for $k = 1, \ldots, N-1$ with boundary condition $a_0 = 0$. The solution, parametrized by a_N, is given by

$$a_k = \frac{k}{N} a_N - 2k(N-k)b, \quad \text{for } k = 1, \ldots, N. \tag{8}$$

Type a_N is indifferent between sending the cheap talk message corresponding to (a_{N-1}, a_N) and the hard signal:

$$-b^2 - c = -\left(a_N + b - \frac{a_{N-1} + a_N}{2}\right)^2.$$

Using equation (8) for $k = N-1$, this becomes

$$-b^2 - c = -\left(\frac{a_N}{2N} + Nb\right)^2.$$

Total differentiation with respect to b and a_N yields,

$$\frac{da_N}{db} = -2N^2 \frac{a_N + 2(N^2-1)b}{a_N + 2N^2 b}. \tag{9}$$

Note that an infinitesimal increase in b leads to a small decrease in a_N.

Differencing (8) for k and $(k-1)$ yields

$$a_k - a_{k-1} = \frac{1}{N} a_N + 2b(2k - 1 - N). \tag{10}$$

Differentiating this in b and using (9) for da_N/db yields

$$\frac{d(a_k - a_{k-1})}{db} = -2N \frac{a_N + 2(N^2-1)b}{a_N + 2N^2 b} + 2(2k - 1 - N).$$

The first term is maximized (minimized in absolute value) at $a_N = 0$, while the last term is maximized at $k = N$. Therefore,

$$\frac{d(a_k - a_{k-1})}{db} < -2\frac{(N^2-1)}{N} + 2(N-1) = 2(N-1)\left(1 - \frac{N+1}{N}\right) < 0. \qquad (11)$$

We conclude that all cheap-talk intervals shrink as b increases, provided that the intervals were non-degenerate.

In equilibrium, the Receiver's payoff (the expected quadratic difference between y and θ where y equals the mean of θ given the information revealed by the Sender) is just the negative of the variance of θ conditional on knowing which cheap-talk interval it belongs to (the Receiver has zero loss when hard signal is sent). Starting from a non-degenerate partition, all cheap-talk intervals shrink in response to an infinitesimal increase in b, therefore the Receiver's expected payoff increases.□

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