Online Appendix for Eső and Galambos (2012), "Disagreement and Evidence Production in Strategic Information Transmission"

Example 2

This example demonstrates that Condition M can fail in our model without further assumptions, and that as a result there can be several equilibria featuring the same number of actions induced through cheap talk.

Let $U^R(y,\theta) = -(y-\theta)^2$ and $U^S(y,\theta) = -(y-\theta-0.01)^2$, that is $b = 0.01$; assume the cost of hard information is $c = 0.01$. Let θ have pdf

$$f(\theta,\varepsilon) = \begin{cases} 5.5 & \text{if } \theta < 0.1-\varepsilon \\ 3 - \frac{5}{2\varepsilon}(\theta - 0.1) & \text{if } 0.1-\varepsilon \le \theta \le 0.1+\varepsilon \\ 0.5 & \text{if } \theta > 0.1+\varepsilon, \end{cases}$$

where $\varepsilon > 0$. The density is continuous and weakly decreasing, so the cdf of θ is continuously differentiable and concave. As $\varepsilon \to 0$, $f(\theta,\varepsilon)$ converges to a density that is a discontinous step function. We use the limiting density

$$f(\theta) = \begin{cases} 5.5 & \text{if } \theta < 0.1 \\ 0.5 & \text{if } \theta \ge 0.1 \end{cases}$$

for the calculation of expected values, which is arbitrarily precise for ε sufficiently small.

Condition M fails

Suppose the first cutoff is $a_1 = 0.09$. For the Sender type at 0.09 to be indifferent between the Reciever's optimal actions $y_1 = \mu(0, 0.09) = 0.045$ and $y_2 = \mu(0.09, a_2)$, it must hold that $(0.045 + y_2)/2 = 0.09 + 0.01$. Thus $y_2 = 0.155$. Then a_2 is determined by the condition that $y_2 = 0.155$ is the expectation of θ given that it falls between 0.09 and a_2:

$$\frac{5.5 \times 0.01 \times 0.095 + 0.5(a_2 - 0.1)(0.5a_2 + 0.05)}{5.5 \times 0.01 + 0.5(a_2 - 0.1)} = 0.155.$$

We solve this to get $a_2 = 0.2823774$.

Suppose the first cutoff is $a'_1 = 0.1$. Sender type 0.1 is indifferent between the Reciever's optimal actions $y'_1 = \mu(0, 0.1) = 0.05$ and $y'_2 = \mu(0.1, a'_2)$, if $(0.05 + y_2)/2 = 0.1 + 0.01$. Thus $y'_2 = 0.17$. Then a'_2 is determined by the condition that $y'_2 = 0.17$ is the expectation of θ given that it falls between 0.1 and a'_2. Since the pdf is constant on this interval, we get $a'_2 = 0.24$.

Thus we find $a'_1 > a_1$ and $a'_2 < a_2$, contradicting Condition M.

Equilibria

First calculate an equilibrium when the first partition ends at $a_1 < 0.1$. The Receiver's response to the first partition is $y_1 = a_1/2$. To make sure that the indifference condition for a_1 holds, the Receiver's response to the second partition will have to be

$$y_2 = 2(a_1 + 0.01) - y_1.$$

Then to find the upper end of the second partition element we solve

$$\frac{5.5 \times 0.5(0.1 - a_1)(0.1 + a_1) + 0.25(a_2 - 0.1)(a_2 + 0.1)}{5.5(0.1 - a_1) + 0.5(a_2 - 0.1)} = y_2$$

to get

$$a_2 = 0.02 \left[1 + 75a_1 + \sqrt{-149 + 6550a_1 - 49375(a_1)^2}\right].$$

We now solve for a_1 using the condition that type a_2 is indifferent between the cheap-talk induced action y_2 and sending the hard signal:

$$(a_2 + 0.01 - y_2)^2 = 0.01^2 + 0.01.$$

This equation is solved by $a_1 = 0.0352302$ (and has another solution as well), which yields $a_2 = 0.163344$.

Now calculate an equilibrium such that the first partition ends at $a'_1 > 0.1$. The Receiver's response to the first partition is

$$y'_1 = \frac{0.05 \times 0.55 + 0.25(a'_1 - 0.1)(a'_1 + 0.1)}{0.55 + 0.5(a'_1 - 0.1)}.$$

To make sure that the indifference condition for a'_1 holds, the Receiver's response to the second partition will have to be

$$y'_2 = 2(a'_1 + 0.01) - y'_1.$$

Then to find the upper bound of the second partition element we use the condition that y'_2 has to be the Receiver's optimal response when the Sender's type is in $[a_1, a_2]$:

$$\frac{0.25(a'_2 - a'_1)(a'_2 + a'_1)}{0.5(a'_2 - a'_1)} = y'_2.$$

This yields

$$a'_2 = 1.04 - \frac{0.55}{0.5 + 0.5a'_1} + 2a'_1.$$

We now solve for a'_1 using the condition that type a'_2 is indifferent between the cheap-talk induced action y'_2 and sending the hard signal:

$$(a'_2 + 0.01 - y'_2)^2 = 0.01^2 + 0.01.$$

This equation is solved by $a'_1 = 0.121674$, which yields $a'_2 = 0.302672$.

Thus we found a second equilibrium with two cheap-talk induced actions and hard signaling.

Example 3

The purpose of this example is to demonstrate that in the uniform-quadratic specification (uniform θ, quadratic loss), when the bias is strictly increasing, it is possible that in equilibrium, a pointwise increase in the bias leads to some cheap-talk partition elements increasing.

Assume that θ is uniform on $[0, 1]$, $y^R(\theta) = \theta$, $y^S(\theta) = \theta + b(\theta, e)$, where $b(\theta, e) = \theta^2/5 + e$ depends on parameter e that we will use to increase bias pointwise. Both players have quadratic loss functions, and the cost of a hard signal is $c = 0.05$.

Suppose the equilibrium mapping (from Sender types to Receiver actions) is characterized by three intervals of Sender types that induce three different actions via cheap talk, and a top partition element in which hard signals are sent. To find the equilibrium partition cutoffs, $0 < a_1 < a_2 < a_3 < 1$, we solve the following indifference conditions:

$$\begin{aligned} a_2 &= 2a_1 + 4b(a_1, e) \\ a_3 &= 2a_2 - a_1 + 4b(a_2, e) \\ b(a_3, e)^2 + 0.05 &= \left[a_3 + b(a_3, e) - \frac{a_2 + a_3}{2}\right]^2. \end{aligned}$$

The first equation says that a_1 is indifferent between inducing $a_1/2$ and $(a_1 + a_2)/2$; the second one is that a_2 is indifferent between inducing $(a_1 + a_2)/2$ and $(a_2 + a_3)/2$, and the last one expresses that a_3 is indifferent between inducing $(a_2 + a_3)/2$ via cheap talk and revealing a_3 via a hard signal, at cost $c = 0.05$.

For various values of the bias parameter e, the following equilibrium partitions result:

e	a_1	a_2	a_3	$a_3 - a_2$
0.0001	0.171154	0.366144	0.668782	0.302639
0.0002	0.170768	0.365664	0.668329	0.302665
0.0003	0.17038	0.365184	0.667876	0.302692
0.0004	0.169993	0.364704	0.667422	0.302718
0.0005	0.169605	0.364223	0.666968	0.302745

Note that each cutoff point is decreasing in e. As the bias increases, more types send hard signals (the length of the highest partition element is increasing). However, the length of the third equilibrium partition element (in which cheap-talk messages inducing $(a_2 + a_3)/2$ are sent) is increasing as well. This cannot happen in the specification with a constant Sender bias, as shown in Proposition 4 of the paper.