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Image Schemas in Clock-Reading: Latent Errors and Emerging Expertise

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An embodied view of mathematical cognition needs to account not only for how we use our bodies to think and communicate mathematically but also how our bodies equip us to conceive of mathematical ideas. Research in cognitive semantics claims that the human conceptual capacity rests on a foundation of image schemas: topological patterns of spatial relations and simple dynamics abstracted from sensorimotor experience. A cognitive ethnography of elementary mathematics lessons in clock-reading reveals different image schemas used to read the hour for landmark times *(three o'clock)*, relative times *(a quarter past three)*, and absolute times *(three fifteen)*. When considered together with the sequence of learning, the image schema analysis predicts the most common error observed in children's time-telling while also revealing the source of latent errors that lead to sudden breakdowns. Comparison of the image-schematic structure of different levels of sophistication in reading "quarter past" times illustrates how enriched image-schematic structure interrelates different aspects of time-telling, supporting flexible performance in novel situations. Relating visible actions to these aspects of ongoing conceptualization will help us provide a more complete account of embodied mathematical cognition, teaching, and learning.

INTRODUCTION

A focus on the embodied aspects of mathematical cognition leads directly to considering how the body is used in mathematical reasoning, in expressing mathematical ideas, and, of particular interest to the learning sciences, in teaching and learning mathematics in educational settings. Yet there may be an even more fundamental way in which mathematical cognition is embodied: in the bodily basis of the conceptual system we use to think mathematically. Research in cognitive semantics over the last three decades provides evidence that the human conceptual system is founded on image schemas, basic patterns of topological relations and simple dynamics that derive from regularities in sensorimotor experience. When we conceptualize mathematical ideas, the image-schematic structure of the conceptualization provides an embodied basis for inferences, supporting mathematical reasoning and problem-solving.

In this article, I examine the role of image schemas in an everyday cognitive activity that receives considerable attention in the early elementary (K-3) mathematics curriculum: reading the time on an analog clock. Telling time is an essential life skill and a fine example of the cognitive complexities involved in learning to perform a seemingly mundane activity. Successful clock-reading calls upon knowledge of numbers, shapes, fractional parts, geometric relations, methods of counting, time divisions, and other mathematical concepts, all of which must be successfully coordinated with structures on the clock face to render different types of time readings. In what follows, I describe image schemas essential to reading the hour correctly for different ways of naming times, such as three o'clock, a quarter past three, and three fifteen. Then I use this analysis to explain how the most common error in children's clock-reading follows directly from the image schemas and the order in which different times are learned. Finally, I illustrate how enriched image-schematic structure in the conceptualization of a clock time supports the flexible performances that are emblematic of understanding and adaptive expertise.

IMAGE SCHEMAS IN MATHEMATICAL COGNITION

Mathematics is widely considered to be a pure and powerful form of human reasoning. And yet mathematical thinking, like all human thinking, must be grounded in the human conceptual system. Drawing on decades of research in cognitive semantics, Lakoff and Núñez (2000) present a detailed analysis of the cognitive foundations of arithmetic, algebra, logic, and set theory, arguing that each is ultimately grounded in embodied experience. They use their analysis to explain how the human cognitive system is able to conceptualize infinity, real and transfinite numbers, continuous and discontinuous functions, and other mathematical ideas. Central to Lakoff and Núñez's embodied account of mathematical cognition is the image schema.

The term "image schema" was introduced in 1987 in books by the linguist George Lakoff and the philosopher Mark Johnson (Lakoff, 1987; Johnson, 1987). In the preface to his book, Johnson defines an image schema as "a recurring dynamic pattern of our perceptual interactions and motor programs that gives coherence and structure to our experience" (xiv). These regular patterns in sensorimotor experience form the basis for our earliest concepts, providing a foundation for the human conceptual system (Mandler, 2004). Hampe (2005) offers this summary of the characteristics of image schemas compiled from Lakoff and Johnson's original works:

– Images schemas are *directly meaningful* ("experiential"/ "embodied"), *preconceptual* structures, which arise from, or are grounded in, human recurrent bodily movements through space, perceptual interactions, and ways of manipulating objects.

> – Image schemas are highly *schematic* gestalts which capture the structural *contours* of sensory-motor experience, integrating information from multiple modalities.
>
> – Image schemas exist as *continuous* and *analogue* patterns *beneath* conscious awareness, prior to and independently of other concepts.
>
> – As gestalts, image schemas are both *internally structured*, i.e., made up of very few related parts, and highly *flexible*. This flexibility becomes manifest in the numerous transformations they undergo in various experiential contexts, all of which are closely related to perceptual (gestalt) principles. (pp. 1-2; emphasis in original)

Among the earliest identified image schemas were PART-WHOLE, CENTER-PERIPHERY, LINK, BALANCE, CONTAINMENT/CONTAINER, and PATH/SOURCE-PATH-GOAL, as well as the force schemas ENABLEMENT, BLOCKAGE, COUNTERFORCE, ATTRACTION, COMPULSION, DIVERSION, RESTRAINT, and REMOVAL OF RESTRAINT (Hampe, 2005, p. 2). As this list suggests, image schemas are thought to be basic topological patterns of spatial relations and dynamics that recur in our sensorimotor experience as embodied beings perceiving and acting in the physical world. They are important because, in Johnson's words, they "help to explain how our intrinsically embodied mind can at the same time be capable of abstract thought. As patterns of sensory-motor experience, image schemas play a crucial role in the emergence of meaning and in our ability to engage in abstract conceptualization and reasoning that is grounded in our bodily engagement with our environment" (2005, p. 15; see also Lakoff, 1990). Put simply, image schemas "make it possible for us to use the structure of sensory and motor operations to understand abstract concepts and draw inferences about them" (Johnson, 2005, p. 24). From this perspective, abstract thought is a form of embodied cognition, depending crucially on the image-schematic structure of conceptualizations.

An elementary mathematical example, adapted from the more detailed analyses in Lakoff & Núñez (2000), is the contrast between different ways of conceiving of addition and subtraction. One way to conceive of these arithmetic operations is in terms of collections of objects: putting collections together (adding numbers) or taking a smaller collection from a larger collection (subtracting a smaller number from a larger number). Here the sensorimotor experiences of manipulating objects and perceiving groupings provide image-schematic structure that frames the conceptual operations of arithmetic, including such properties as magnitude, stability of results, inverse operations, commutativity, associativity, and so on (pp. 54-60). The image-schematic structure inheres in the mathematical conception even in the absence of any connection to the world of physical objects. With this conception, however, the notion of subtracting a larger number from a smaller number is nonsensical: it is impossible to extract a larger collection from a smaller one. On the other hand, if addition and subtraction are conceived of in terms of directional motion along a path, a number line, then subtracting a larger number from a smaller number becomes sensible: moving to the left past the origin, where negative numbers take on a clear meaning (pp. 71-74). Differences in the image-schematic structure of these conceptions of addition and subtraction yield fundamentally different meanings: a number as a collection of objects vs. as a location, a magnitude as the size of a collection vs. as a distance, the operations of addition and subtraction as placing and removing objects vs. as moving in a specified direction, zero as absence vs. as origin, and so on. The example illustrates how the same named entity can be conceptualized in different ways, but more importantly, it shows how differences in the image-schematic structure of these conceptualizations support different inferences. Inferences that are readily apparent in one conceptualization may be invisible or unsupportable in another.[1] These points will bear directly on our analysis of image schemas in clock-reading and the implications that follow.

SOURCE OF THE DATA

The analysis of clock-reading presented here is based on a cognitive ethnography of elementary time-telling instruction undertaken as dissertation research for a doctoral degree in cognitive science (Williams, 2004). Cognitive ethnography is the study of how cognitive activities are accomplished in real-world settings (Hutchins, 2003; Williams, 2006b). Its aim is to provide an ecologically valid account of human cognition as an embodied process embedded in culturally constituted activity. Its method consists of detailed micro-analysis of recordings of situated cognitive activity—reasoning, decision-making, and problem-solving, often involving multiple participants and/or various tools and representational artifacts. Ethnographic evidence gathered from participant observation, interviews, artifact analysis, and so on, is used to warrant interpretations of the video data. This work is undertaken with the intent of explaining how cognitive activities are actually accomplished, what resources are brought to bear, how these are coordinated with one another, what changes or adaptations occur, and so on. The analysis of activity provides a basis for inferring what must be happening in the internal cognition of individuals for the overall system to function.

For the cognitive ethnography of time-telling instruction, data were gathered in two settings: in 1st, 2nd, and 3rd grade classes in an inner-city church school in San Diego, California, with a working class, mixed-ethnic student population; and in a 2nd grade class in a private elementary school in La Jolla, California, with a wealthier, predominantly Caucasian student population. In each of the classes I observed mathematics lessons at various times throughout the 2002-2003 school year and made digital video recordings of lessons related to time-telling, collected materials used, observed student learning activities, and asked teachers and students about their work. I also conducted a series of individual interviews with 3rd grade students where I posed various time problems using both analog and digital clock displays, from naming the time to determining the interval to some past or future time. Here I asked students to explain the reasoning behind their solutions, and I observed their talk and gestures for clues to their conceptualization. The interviews were loosely structured, probing the limits of the student's time-telling, problem-solving, and explanatory abilities.

Episodes of classroom instruction in time-telling and student interviews were the primary sources of data for the present analysis of image schemas in clock-reading.

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[1] A related example suggested by an anonymous reviewer is how object collections support the conceptualization of discrete whole numbers (counting numbers) while the number line supports the conceptualization of real numbers in their limitless density.

Details about method, including sample lesson episodes, transcripts incorporating talk and gestures, and cognitive linguistic analysis, can be found in Williams (2004, 2006b).

IMAGE SCHEMAS IN CLOCK-READING

In our consideration of clock-reading we will focus on three image schemas—PROXIMITY, CONTAINER, and SOURCE-PATH-GOAL—and the role they play in time-telling. The PART-WHOLE and CENTER-PERIPHERY image schemas are also involved in reading the clock dials: hours, quarter-hours, minutes, or seconds. Discussion of the dials and the role played by PART-WHOLE and CENTER-PERIPHERY structure in clock-hand pointing can be found in chapter 4 of Williams (2004). Other image schemas, such as CYCLE, are clearly of central importance to time-telling but go beyond the scope of the present paper.

The three spatial image schemas to be discussed are shown in Figure 1. Following a tradition in cognitive linguistics dating back to Langacker (1987), the object that has focal prominence—that is foregrounded as the locus of attention—is labeled the *trajector* (TR), while the object of lesser prominence that the trajector stands in relation to is labeled the *landmark* (LM). In the PROXIMITY image schema, the trajector is located in the vicinity of the landmark, in what—if the landmark were animate—might be the landmark's immediate domain of control. Proximity is an embodied sense of nearness, of impending interaction or near-unity, which diminishes with increasing distance from the landmark, giving it a scalar quality. In the CONTAINER image schema, the trajector is located within a two- or three-dimensional region of space bounded by the landmark. We say that the trajector is contained within the landmark or that the landmark contains it. The boundaries of the landmark may be demonstrably real, as when a marble is contained in a jar, or they may be conceptually projected, as when a bee flies into the garden. The SOURCE-PATH-GOAL image schema provides the structure of a basic motion event: the trajector begins moving from a location designated as the source, passes through a series of contiguous locations that constitute a path, and completes its motion at a location designated as the goal. At any given moment, the trajector occupies some position along the path from source to goal. The path may be factive—the actual path of a moving object—or it may be fictive (Talmy, 1996/2000), as in the sentence, "The fence runs from the house to the barn," where the fence does not move, but motion is nevertheless experienced as subjective scanning along the fence in the visual scene or in the mind's eye. Each of these three image schemas—PROXIMITY, CONTAINER, and SOURCE-PATH-GOAL—plays a pivotal role in a particular type of time-telling, as described below.

<Insert Figure 1 here>

Landmark Time

Landmark time is the most basic time, the earliest in the development of time-telling practices and the first learned. It is the occurrence of a reference event: the intersection of a shadow with a mark on a floor, wall, stick, or sundial, or of a clock hand with a tick mark or number on the clock face. In early time-telling, the intersection of a shadow with an inscribed mark corresponded to what will here be called a *landmark hour*, one of the twelve equally spaced reference events from sunrise to sunset, mid-day being the 6th hour. The duration between these reference events varied with the seasons, so that conceptually an hour was unlike the clock hour we know today (Williams, 2004). Many centuries later, mechanical clocks established hours of fixed duration, leaving only noon anchored to the sun. Noon became the new origin/endpoint of the 12-hour cycle, its significance signified by the clock hands pointing directly upward. The later innovation of time zones and daylight saving time disrupted even this correspondence. Landmark time survives today in "o'clock" times, literally time "on the clock" as opposed to solar time.[2] The "o'clock" times are the first times that children learn to read on the analog clock and to associate with life activities like mealtimes and bedtimes, giving early meaning to clock time.

When children begin to read landmark hours, they may recognize right away that the long hand points up to the *12*, but to read the time they need to focus their attention on the short hand and the number it points to, as shown in Figure 2. Reading the landmark hour involves the PROXIMITY image schema closely associated with pointing in human discourse: a person points to draw attention to a referent, cuing the addressee to scan along the trajectory of the point in search of a suitable reference object; where multiple relevant objects are present, the one nearest the trajectory of the point is taken as the most likely referent. On the clock face, when the long hand is at the *12*, the short hand points toward a particular numeral[3]; this is the taken as the number of the landmark hour. Teachers reinforce this association by telling students that the hour is the number the short hand "points to." If the long hand is in the vicinity of the *12*, the landmark hour is taken as the number closest to the trajectory of the short hand's point, using the PROXIMITY image schema conventionally associated with pointing. Once this number has been selected (say *3*, for example), the time is conventionally reported as "<*number*> o'clock" ("three o'clock") or, in the case of approximation, as "about <*number*> o'clock" ("about three o'clock"). We see that reading "o'clock" times retains the historic sense of naming distinct landmarks and that proper reading of the landmark hour depends on the PROXIMITY image schema.

<Insert Figure 2 here>

Relative Time

Once landmark times are established, a child quickly gains a sense of the long hand approaching the *12* or moving past the *12*, so that it can be "almost <*number*> o'clock" or "just after <*number*> o'clock," and so on. This provides the start of relative time-telling.

Relative time is a historical elaboration on landmark time, first with quarter hours and later using minutes once the hour was divided into 60 "small parts," as described by

[2] More about the cultural history of time-telling can be found in Barnett (1998); the relation between this history and changing conceptualizations of time is explored in chapter 3 of Williams (2004).

[3] Interpreting a clock hand as pointing involves additional image-schematic structure discussed in Williams (2006a).

Barnett (1998). Relative time states the time in relation to a landmark, as either past or till the landmark hour. Examples are “a quarter to one” or “ten past three.” Here, because the landmark is used as a cognitive reference point, we will call it a *reference hour* to distinguish its use in relative time-telling. To read relative time, both clock hands must be attended to and read in relation to one another. For the long hand, attention focuses on reading the relation between the hand’s current position (where it points on the dial; for brevity, we will simply refer to the tip of the long hand as the trajector) and its reference position at the top of the hour when it points to the *12* (the landmark). When the relation is read as “past,” the reference hour is the preceding landmark time (“<*number*> o’clock”); when it is “till,” the reference hour is the upcoming landmark time.

While proximity still plays a role in choosing whether to read the relation as “past” or “till,” properly reading the reference hour requires a conceptual shift to the SOURCE-PATH-GOAL image schema as shown in Figure 3. Recall that the SOURCE-PATH-GOAL image schema structures the conceptualization of a full path of motion. A clock hour begins with the tip of the long hand (the trajector) at the top of the clock on the *12*, which is the SOURCE. As the hour unfolds, the trajector moves steadily around the dial clockwise, along a predefined PATH, until it reaches the *12* again; this same location is now construed as the GOAL. On an actual clock, the long hand and short hand do not move independently. Instead, a geared mechanism locks in the invariance essential to time-telling: as the tip of the long hand ($\mathrm{TR}_{\mathrm{long}}$) moves from the *12* ($\mathrm{SOURCE}_{\mathrm{long}}$) clockwise around the clock to the *12* ($\mathrm{GOAL}_{\mathrm{long}}$), the tip of the short hand ($\mathrm{TR}_{\mathrm{short}}$) moves around the clock clockwise from one number ($\mathrm{SOURCE}_{\mathrm{short}}$) to the next ($\mathrm{GOAL}_{\mathrm{short}}$). The movement of the long hand around the dial is linked both mechanically and conceptually to the movement of the short hand from one number to the next. The link between these two paths of motion means that for “past” time readings the reference hour must be the *source* of the short hand motion ($\mathrm{SOURCE}_{\mathrm{short}}$), as in “a quarter past three,” whereas for “till” readings it must be the *goal* of motion ($\mathrm{GOAL}_{\mathrm{short}}$), as in “a quarter till four.” This is true regardless of proximity: “forty minutes past ten” is referenced to the *10* even though the short hand is closer to the *11*. In practice, people commonly refer times to the nearest landmark, reporting times during the first half of the hour as “past” and times during the second half of the hour as “till.” If a clock reader always follows this convention, then simply reporting the nearest number—relying on proximity—will produce correctly named times even where the reader lacks awareness of the conceptual relationship between the two hands, a problem to be discussed below. Only when the long hand is furthest from the *12*—when it points to the *6* and is therefore equidistant on the “past” and “till” sides—does it become impossible to rely on proximity. Here linguistic convention dominates: the British report the time as “half past three” while the Germans report it as “halb vier” (“half four”).[4]

[4] Actually, the situation is somewhat more nuanced. Along with “halb vier” (“half four”), German speakers also report a quarter past three as “ein Viertel vier” (“one-quarter four”) and a quarter till four as “drei Viertel vier” (“three-quarters four”). This way of conceptualizing the time combines SOURCE-PATH-GOAL structure (the motion of the long hand through one clock hour) with endpoint focus (reference to the GOAL) and PART-WHOLE structure (reporting the proportion of the PATH that has already been traversed). The hour may also be construed as a container being filled and the long hand as moving along a scale (a SOURCE-PATH-GOAL structure) from empty to full, with its present position marking the current fill level. A clock

<Insert Figure 3 here>

Absolute Time

More accurate mechanisms of time-keeping enabled greater precision in time measurement, supporting the naming of times as a precise number of hours, minutes, and even seconds (literally, “second small parts”). These can be written conventionally in numeric form (e.g., as ‘3:30’). Advances in electronics have made digital time displays so ubiquitous that it has become commonplace to name times precisely as, for example, “three fifty seven,” furthering a trend in America away from relative time-telling to naming the time precisely in absolute form.

For absolute time, the hour is no longer a landmark or reference point. Instead it is the *current hour*, the first portion of a time stated in units of hours, minutes, and (optionally) seconds. Reading the current hour on the analog clock requires yet a different image schema, the CONTAINER schema, as shown in Figure 4. If the short hand points to a location between *3* and *4*, the current hour is “three,” followed by some number of minutes (and possibly seconds) read using the other clock hand(s), as in “three thirty.” Consider what happens when the time is 3:57: the tip of the short hand is in close proximity to the *4*—almost coincident with it—making *4* clearly the number the short hand points to. But when reading the hour for absolute time, proximity is irrelevant and often quite misleading. As long as the short hand is contained within the region of space bounded by its position at three o’clock and its position at four o’clock, the hour continues to be “three.” Only when the short hand arrives fully at the *4* and begins crossing into the next region does the hour change.

Notice that the landmark “o’clock” times remain present in both relative and absolute time-telling but play different roles. In relative time-telling, the landmarks serve as reference points: the source or goal to which the present time is relationally referenced. In absolute time-telling, the landmarks are boundaries of containers, the interiors of which are conceptually associated with particular hours. Crossing the boundary changes the current hour, making boundary crossing a highly salient event. Despite the salience of boundary crossings, no conceptualization of motion is required to read absolute time: the current hour is determined solely by the location of the trajector in one container or another. And in sharp contrast to relative time, the hours and minutes for absolute time can be read independently without particular regard to the clock hands’ relation to one another. Absolute time is static, identifying the present moment, while relative time is dynamic and relational, expressing the interval between the present moment and some past or future reference time.

<Insert Figure 4 here>

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reader need not conceptualize the situation this richly in order to name the time correctly, a topic taken up in the section on emerging expertise.

Comparison of Time Formats and Image Schemas

The time formats and their associated image schemas are summarized in Table 1. The table shows the close correspondence between the linguistic format for reporting the time and the preferred order of looking at the dials embedded in the clock face, once the clock has been initially glimpsed.[5] For landmark time, only the hour is reported ("five o'clock"), so only the hour dial—the short hand, numerals, and major tick marks—needs to be attended to closely. The clock hand configurations for landmark times soon come to be recognized and named directly, making even the numerals superfluous. For relative time, the long hand is typically focused on first; whether the time interval is read as a half- or quarter-hour or as a span of minutes depends on the position of the long hand and the readings it affords, as well as cultural convention. The long hand positions at *6*, *3*, and *9* become recognized as landmarks for "half past," "quarter past," and "quarter till," respectively, and with practice other five-minute times come to be recognized and named directly as "ten past," "twenty past," and so on (later even as "ten till," "twenty till," etc.), creating an array of secondary and tertiary landmarks. For absolute time, children who are learning to tell time typically read the hour first and then count by fives and/or ones to read the minutes; more experienced time-tellers count the minutes from the nearest five-minute mark (in studies of adult by Case, Sandieson, and Dennis, 1986). Reading the dials in a different order and then manipulating the information into another format is possible but places additional loads on memory and processing. This is perhaps most likely to occur when there is some conflict between the demands of the situation, the affordances of the particular clock hand configuration, and/or habitual ways of telling the time.

<Insert Table 1 here>

Most importantly for our purposes, Table 1 shows the use of three distinct image schemas for reading the hour portion of the time in the three different formats: the PROXIMITY image schema for reading the *landmark hour* in landmark time, the SOURCE-PATH-GOAL schema for reading the *reference hour* in relative time, and the CONTAINER schema for reading the *current hour* in absolute time. Because time-telling is a complex skill that takes children years to master, the order in which different forms of time-telling are learned can affect the use and misuse of particular image schemas in clock-reading. This is the subject of the next section.

LATENT ERRORS IN CHILDREN'S CLOCK-READING

In a study of children's clock-reading strategies, the most commonly observed error was misreading the hour, and this error occurred exclusively when the long hand was on the

[5] Eye-tracking studies by Bock, Irwin, Davidson, and Levelt (2003) confirm that participants do indeed alter their order of looking at the clock hands to conform to the requested format for reporting the time. They also found in two of their experiments that a single participant, an American English speaker, persisted in viewing the hour hand first when asked to report relative time, showing both the American penchant for absolute time (confirmed in a separate experiment by Bock et al.) and the ability to mentally manipulate a time reading into a different format.

left side of the clock (Siegler & McGilly, 1989, p. 209). The analysis of image schemas in clock-reading presented above predicts this effect; in doing so, it also reveals a deeper concern about learning complex skills in mathematics and other domains. We begin by considering the order in which time-telling skills are learned because this lays the groundwork for predicting the pattern of errors.

The Order of Learning to Read Times on an Analog Clock

Children learn time-telling over a period of several years, beginning in the preschool ages and continuing through the elementary grades. Table 2 summarizes findings from three studies of American children's clock-reading at ages ranging from 4 to 10 years (Springer, 1952; Friedman & Laycock, 1989; Siegler & McGilly, 1989). The table shows a consistent pattern of development. Children begin with the landmark hour times, and by age 7 all the children in the studies have mastered "<*number*> o'clock." The next times to be mastered are the secondary landmarks: the half-hour times, which may be read either as "half past <*number*>" or "<*number*> thirty"; the studies do not identify which form was used. Next come the 15-minute times, also without distinction between "quarter past <*number*>" and "<*number*> fifteen," followed closely by the 5-minute times. Last to be mastered are the 1-minute times; by age 10 (the highest age in the studies), the 1-minute times are read with 80% accuracy. In general, the studies make no distinction between relative and absolute time-telling, noting only correctness. More recent work by Bock, Irwin, Davidson, and Levelt (2003) provides evidence of significant cultural differences in preferences for expressing time, with Dutch undergraduates strongly favoring relative time while American undergraduates express time almost exclusively in absolute form. As we have seen in the image schema analysis, differences in the form for reporting the time imply differences in how the reading was accomplished.

<Insert Table 2 here>

Classroom observations from Williams (2004) confirm this general outline while filling in considerable detail. The progression of learning to read clock times observed in the ethnographic study is shown in Figure 5. Although some landmark forms of relative time (half past and quarter past/till) were introduced early, an emphasis on relative time came only after absolute time had been reasonably mastered. The progression started with reading "o'clock" times and associating these with daily activities (mealtimes, bedtime, etc.). Half-hour times were introduced next, and although motivating explanations were offered—the short hand being halfway between two numbers (for "half past"), or 30 minutes being half an hour (for "<*number*> thirty"), etc.—students were expected to name the times directly, without counting on the clock, as secondary landmarks to the "o'clock" times. Dividing the clock face into quarters appeared at all levels from 1st through 3rd grade; here again, motivating explanations were initially provided and then followed by practice naming times directly as "quarter past" or "quarter till," making these tertiary landmarks. Calling these landmarks by their absolute names—"<*number*> fifteen" or "<*number*> forty-five"—involved counting by fives on the clock, which led naturally to naming other 5-minute times; with practice these also come to be recognized directly as landmarks. Counting on by ones from a known or established 5-minute time produced the 1-minute times. All of these were practiced at intervals during math lessons from kindergarten to 3rd grade.

In 3rd grade, the teacher led the students through a review of the meaning of “quarter past” and “quarter till” as she introduced other forms of relative time-telling, including time till the upcoming hour (e.g., “twenty till <*number*>”), time past or till the hour, and time till some future reference time (say, an appointment at 10:15). “Time till” problems were solved by reading the absolute time and subtracting the minutes from 60; students were encouraged to check their answers by counting by fives and ones from the *12* back (counterclockwise) to the current position of the long hand. Students had much more difficulty solving time problems that crossed the hour boundary, such as determining the number of minutes from 1:35 to 2:10. Successful students decomposed these problems into time till the hour plus time after the hour, or they counted around the clock from one position to another when both were 5-minute times; some recognized common intervals such as the half-hour (180-degrees across) from 9:45 to 10:15.

<Insert Figure 5 here>

Overgeneralization of Proximity for Reading the Hour

Given this background, how does the image schema analysis explain the most pervasive error in children’s clock-reading? Siegler and McGilly (1989) found that fully one-third of 8- and 9-year-olds’ clock-reading errors were misreading the hour when the long hand is on the left side of the clock, for example, misreading “three fifty” as “four fifty.” The cause appears to be overgeneralization of the PROXIMITY image schema commonly associated with pointing. The explanation based on image schema analysis goes like this: Children learn to read the landmark “o’clock” times first and are taught that the hour is the number the short hand “points to.” Pointing in everyday life is associated with proximity: the referent is the first suitable object encountered when scanning along the trajectory of the point—in other words, the one in closest proximity (Williams, 2006a). Children are therefore likely to continue to attend to proximity when they view the short hand while reading other types of clock time.

What happens when children overgeneralize the PROXIMITY schema to reading absolute and relative times? The answers are shown in Figures 6 and 7. Each figure shows the use of the erroneous PROXIMITY image schema on the left and of the correct image schema (CONTAINER or SOURCE-PATH-GOAL) on the right. The hour portion of each reading is boxed, with a diagram showing how the image schema produces a name for the hour and whether that name is correct (incorrect names are marked with an “X”). Figure 6 shows overgeneralization of the PROXIMITY schema when reading absolute time, the form preferred by Americans and emphasized in early time-telling instruction. Using the wrong image schema, children name absolute times correctly when the long hand is on the right side of the clock (three fifteen) and incorrectly when the long hand is on the left (four forty-five). The image schema analysis accounts for the sudden emergence of clock-reading errors in the latter half of the hour, and it predicts that these errors should increase as the time approaches the upcoming hour, bringing the short hand into closer proximity with the next number on the dial. Figure 7 shows a similar overgeneralization of the PROXIMITY schema to reading relative time. In this case, times can be named correctly using the wrong image schema whether the long hand is on the right (quarter past three) or left (quarter till four) as long as children follow the convention of always referencing the time to the nearest hour (past or till). Here apparently correct performance masks serious misunderstanding. With overgeneralization of the PROXIMITY schema, a less conventionally named time such as “fifty past ten” would seem nonsensical because without SOURCE-PATH-GOAL image-schematic structure there is no conceptual relation linking clock hand motions to their respective sources and goals.

<Insert Figures 6 and 7 here>

What is striking about Figures 6 and 7 is that in three of the four conditions (absolute-right, relative-right, relative-left), children name times correctly using the wrong image schema, while in only one condition (absolute-left) do they fail. This is 75% accuracy, widely considered an acceptable standard of performance, one that would earn a grade of “C” (satisfactory) on the A-F grading scale used in most American schools. The children’s apparent competence masks a fundamental conceptual error that can persist undetected and unabated.

The problem of being drawn into the natural association with proximity is readily apparent in classroom observations of time-telling lessons. Figure 8 shows a brief excerpt from a 1st-grade lesson on reading “six forty-five.” Here the clock face shows the long hand on the *9* and the short hand nearly touching the *7*. The teacher says, “It’s not *seven* forty-five,” and as she says “not *seven*” she traces a curved trajectory from the tip of the short hand to the *7*, explicitly depicting the proximity associated with pointing that she is verbally negating. She goes on to give the reason: “cause it’s *not* on the seven yet,” which would be the landmark that marks the boundary crossing into the next hour region. She reiterates the proper time, “it’s *six* forty-five,” and as she says “it’s *six*” she traces a line from the center of the clock down to the *6*, the defining boundary of the *6* region. Her two quick gestures enact the key conceptual distinction in how the children should view the clock face: which image-schematic structure should be imposed there to produce the correct hour reading. The teacher’s gestures and talk negate proximity while highlighting the boundaries of the region that contains the short hand; so long as the short hand is anywhere in the space between *6* and *7*, the current hour is “six.” In this way, the teacher guides the learners in seeing the clock in a particular way that supports a particular form of time-telling. More detail about how the teacher’s gestures and talk guide learners’ conceptualization can be found in Williams (2008a, 2008b), including how gestures add image-schematic structure to the conceptualization.

<Insert Figure 8 here>

Overgeneralization of Containment for Reading the Hour

It was noted earlier that Americans favor absolute time and that classroom observations confirm this finding. If children are learning absolute time-telling thoroughly before devoting much attention to relative time, we might expect some interference from the already-mastered form. We have already seen that when children read absolute times in the latter part of the hour, they are encouraged to focus on the boundaries that separate one hour region from another. What happens if they overgeneralize this form of hour reading into relative time? The answer is shown in Figure 9, with improper use of the CONTAINER image schema on the left and proper use of the SOURCE-PATH-GOAL image schema on the right. Again we see that when the wrong image schema is used, the time is named correctly when the long hand is on the right (quarter past three) and incorrectly when it is on the left (quarter till three).

<Insert Figure 9 here>

If we put all of these image schema overgeneralization errors together in Table 3, a clear pattern emerges. In the table, "yes" indicates correctly named times while "no" indicates incorrectly named times. The labels "error" and "latent error" identify conceptual misunderstandings, but while those labeled "error" lead to incorrectly named times, those labeled "latent error" go unnoticed. What is striking is that in two thirds of the cases where a systematic conceptual error is present—where the conceptualization of the situation is fundamentally flawed—the error persists undetected. In only one third of the cases does the conceptual error result in an incorrectly named time, and this happens to occur only when the long hand is on the left side of the clock. Whether children overgeneralize proximity from its close association with pointing, which seems likely, or containment from extensive practice reading absolute times, the result is the same: an error rate of approximately 25% overall (perhaps less if proximity errors emerge only when the long hand is quite close to the upcoming number), with all errors occurring when the long hand is on the left side.

<Insert Table 3 here>

When children read the hour using the wrong image schema, success in time-telling can break down suddenly, triggering a search for the cause of error in the particulars of the situation. What is it about the second half of the hour that makes it harder to read than the first? Why does crossing the *6* make a competent time-teller start to fail? Perhaps students have more difficulty with larger numbers, or perhaps they experience confusion over the differently named hours for current and "till" times. These are reasonable questions and hypotheses. The problem is that they direct us to look for the *source* of error in circumstances that may simply have *revealed* the error, which turns out to be systematic. This type of conceptual error can pervade the child's time-telling while remaining largely unnoticed.

In summary, the analysis of image schemas in clock-reading offers a parsimonious explanation for the clock-reading errors observed in the Siegler and McGilly (1989) study. It also highlights the effects of the order of learning, demonstrating how guidance at one stage produces conceptual difficulties at later stages, and it explains how latent conceptual errors lead to sudden breakdowns. The advice so essential to reading landmark time, that the hour is the number the short hand points to, reinforces the natural focus on proximity that must later be ignored and replaced by a sense of the short hand being contained within a bounded region to read absolute times and by awareness of the sources and goals of clock hand motion to read relative times. Early learning sets the stage for later learning, and existing conceptions are used to make sense of new experiences. In this analysis, we have seen how prior learning can complicate new learning and how successful performance can mask serious misunderstanding.

EMERGING EXPERTISE IN CONCEPTUALIZING CLOCK TIME

What does the image schema account tell us about the conceptual changes associated with emerging expertise in time-telling? We have already seen how becoming expert at reading the different kinds of times requires developing a repertoire of different ways of reading the short hand, each involving a different image schema, and recognizing when each way should be used. Expert time-tellers read the short hand quickly and flawlessly whether they are reporting landmark, relative, or absolute time. Novices develop this ability in stages, with changes in image-schematic structure marking the developmental progression, and with failures to change image schemas leading to misunderstanding and error. But what about conceptual changes associated with reading a single type of time?

Figure 10 provides an illustration of emerging expertise in reading "quarter past" times, based on the cognitive ethnography of time-telling instruction (Williams, 2004). Beginning clock-readers, even when given an explanation for why the time is called "a quarter past," succeed most readily by simply associating the label "a quarter past" with the long hand's orientation: pointing horizontally to the right at the *3*. In a 1st grade lesson introducing "quarter past," for example, the teacher briefly introduced the term "a quarter," showed the students how to divide the clock face into quarters, and then had the students practice naming "quarter past" times with different hours (see Williams, 2006b, for the structure of the lesson). This created a situation similar to when the students first learned to read the landmark "o'clock" times: the students focused their attention on the short hand, concentrated on reading the hour correctly, and treated the long hand's position at the *3* as a prompt for saying "quarter past," just as they learned to say "o'clock" when the long hand pointed to the *12*. The association was reinforced through repetition, and the "quarter past" times were not mixed with other clock times[6] (although students did learn to name the times as "<*number*> fifteen." At this stage, no understanding of the meaning of "a quarter past" was required to perform successfully. This conceptualization is represented by the diagram on the far left in Figure 10.

<Insert Figure 10 here>

Motivating explanations provided in lessons on reading quarter-hour times typically involve dividing the clock face into halves and then into quarters, as happened briefly in the lesson described above. In some classes, children are given paper clock faces and instructed to draw a line from the *12* down to the *6* and then another line from

[6] Students did learn to rename "quarter past" times as "<*number*> fifteen." See Williams, 2008a, for detailed analysis of how the teacher's talk and gestures systematically constructed two different ways to conceptualize the same clock state, each with its own distinct image-schematic structure.

the *9* across to the *3*. When the long hand aligns with any of these dividing lines, it stands in one of the canonical positions that come to be known as “o’clock,” “a quarter past,” “half past,” or “a quarter till.” Children are often instructed to color in the upper right quadrant and label it “a quarter past.” When the long hand points to the *3*, it forms one boundary of this quadrant, the other being the landmark “o’clock” position. Children who associate the label “a quarter past” with this shape and who recognize the shape as the upper-right quarter of a canonically divided circle come to have the somewhat more sophisticated understanding represented by the second diagram from the left in Figure 10. Here the conceptualization incorporates PART-WHOLE image-schematic structure: the clock face is seen as composed of four fractional parts arranged in a standard configuration. This conceptualization is static, involving nothing more than the state of the clock and the arrangement of its parts. The long hand is conceptualized as the boundary of a shape rather than as a moving object.

An even more sophisticated understanding is represented by the third diagram in Figure 10. Here the actual dynamics of clock-hand motion have been added to the conceptualization. The tip of the long hand—the trajector—begins its hourly journey at the *12*, and when it arrives at the *3*, it has traveled one fourth of the way along its path around the clock. In other words, it has traversed one quarter of the path from source to goal, and it is therefore a quarter past the previous hour. When the trajector reaches the *6*, it will be half past the hour. When it reaches the *9*, it will have traversed three quarters of its hourly path around the clock and will have one quarter left to traverse; hence it will be a quarter till the upcoming hour. This depth of understanding comes from superimposing SOURCE-PATH-GOAL image-schematic structure onto the PART-WHOLE structure of the clock quarters. The motion of the long hand around the clock can then be linked conceptually to the motion of the short hand from number to number, binding the different parts of a relative time reading.

This more expert-like conceptualization can be enriched still further, associating path segments with time intervals as shown on the right in Figure 10. The full path corresponds to one hour, which is 60 minutes. This entails a spectrum of clock span and time interval relationships: that one fourth of the path corresponds to 15 minutes, one third to 20 minutes, and so on. A quarter past the hour is therefore equivalent to 15 minutes past the hour. Each movement of the long hand from one number to the next then represents the passage of an additional 5 minutes because each of these movements covers one twelfth of the total 60-minute path. When the long hand points to the *3*, the elapsed time since the previous hour is three times five minutes, providing the conceptual link that explains why we look at the *3* and call it “fifteen.” All of these inferences result from the complex image-schematic structure of the expert conceptualization: linking two paths of clock-hand motion with several part-whole relations that include fractional parts of a circle, path segments as proportions of a full path, and the division of duration into conventional units of hours and minutes in our cultural system of time measurement. If someone is catching a train at 11:05 and the clock displays ten minutes till eleven, then it is a quarter-hour till the departure time. This can be read directly from the span between the present and future positions of the long hand on the clock face, or it can be computed as 10 till the hour plus 5 past the hour, which equals 15. A complex of rich image-schematic structure provides the system of conceptual interrelationships that enables this expert reasoning—reasoning that is unsupported by the simpler conceptualizations to the left of Figure 10.

All of these ways of conceptualizing "a quarter past" yield the correctly named first part of a relative time reading when the long hand points to the *3*. What distinguishes the more sophisticated understandings is their rich array of image-schematic structure, incorporating diverse conceptual elements related to reading clock times—fractional parts of a circle, paths of motion, proportions of paths traversed, number lines, hierarchical units of time, and so on—all richly interconnected to support reasoning in everyday situations and adaptability to novel circumstances. The image schema analysis for "a quarter past" shows how this kind of understanding can emerge invisibly, without failure or disruption in everyday performance. Considered together with the analysis of different image schemas for reading the hour, what emerges is a portrait of expertise as involving more sophisticated conceptualizations with richer image-schematic structure and alternative conceptualizations with different image-schematic structure, with the flexibility to switch conceptualizations guided by a sense of appropriateness that the more sophisticated conceptualizations help to provide.

DISCUSSION

Conceptualizing Mathematical Ideas

In everyday life, mathematics is treated as a collection of conventional methods for solving familiar problems—in short, as procedural. As an intellectual activity, however, mathematics is deeply conceptual: capturing relations and manipulating them, whether to prove or disprove conjectures, to solve real-life problems, or to explore the realm of possibilities. More than any other discipline, mathematics tries to distill entities and relations to abstract form, bleaching out imageable content while preserving image-schematic structure as a basis for reasoning. Yet when called upon to think mathematically, a learner must conceptualize the given situation in a particular way, and the image schemas that structure this conceptualization will determine what inferences are available and what outcomes likely.

It is apparent that mathematical ideas can be conceptualized in more than one way, with differences in image-schematic structure yielding different insights. A line segment can be conceptualized as a set of points (PART-WHOLE) or as the path defined by a moving point (SOURCE-PATH-GOAL), two very different ways of thinking about the same mathematical entity. Mathematical ideas can also be conceptualized with varying degrees of richness or detail: "a quarter past" can be simple orientation, part-whole structure, part-whole with a path of motion, or a complex of part-whole structures and interconnected paths of motion. The rich image-schematic structure of more sophisticated conceptualizations supports a broader range of possibilities for reasoning, exploring relations, and drawing conclusions; in other words, it makes more things seeable and knowable. The image-schematic structure of initial conceptions, by contrast, can be quite impoverished and can even prove misleading when new problems are encountered, as when reading landmark times by proximity disrupts the reading of absolute times where a different way of interpreting the short hand's relation to the clock face is required. Misunderstandings can arise from different conceptualizations of the situation, even where each may be valid in its own right: a teacher and a learner, or two learners, can look at the same configuration of hands on a clock face and "see" something different there, not understanding why the other does not see what they do.

Effects of the Sequence of Learning

Among other things, the present study of image schemas in clock-reading illustrates how well-intentioned advice at early stages of learning can produce misconceptions at later stages, leading to sudden breakdowns in performance. To take another mathematical example: a teacher tells a child that to find the difference between two numbers "you start with the larger number and take away the smaller number," preserving the image-schematic logic of object collections. This turns out to be great advice for the initial learning of subtraction, just as telling the child that the hour is "the number the short hand points to" is great advice for helping the child correctly name "o'clock" times. Later, though, the teacher may be left scratching her head when the child claims that 35 minus 29 equals 14, a type of subtraction error observed in a 2nd grade class while conducting the clock-reading study. Following the teacher's advice, the child has solved the problem in a logically consistent way: 3 minus 2 equals 1, and 9 minus 5 equals 4. This is akin to the child reading 6:45 as "seven forty-five" because the short hand points to the *7*. The source of the breakdown lies in a previous stage of learning where a particular way of conceptualizing the situation produced success at solving the class of problems at hand: single-digit subtraction or reading landmark times. At a later stage, this way of conceptualizing the situation interferes with the conceptualization needed to solve a new class of problems: two-digit subtraction or reading absolute times. This is not to suggest that the initial advice was flawed; it was in fact exactly what was called for at that stage of learning, given the conceptual basis for the learner's present understanding: manipulating object collections (for subtraction) or locating landmarks (for naming clock times). More expert thinking will involve additional image-schematic structure, such as PART-WHOLE relations for place values, or alternative image-schematic structure, such as SOURCE-PATH-GOAL relations for viewing subtraction as motion on a number line. These call for different advice—advice that might contradict an earlier way of thinking. Because learners at various stages of advancement conceive of a situation in diverse ways—with varied richness, complexity, and appropriateness of image-schematic structure—teachers must tailor their advice to each learner's thinking. An utterance or gesture that helps one learner to an insight might do nothing for another or, worse, lead to further confusion. In this regard, instruction is always a highly situated activity.[7]

[7] An anonymous reviewer rightly points out that it is not only the sequence of learning and guidance that shapes the use of particular image schemas but also experience with certain kinds of tools. Children's use of spinners with board games, for example, would provide good support for using the container schema to read the hour. Whereas spinners have clearly demarcated boundaries, color-coded regions, and numeric labels centered in each region, clock faces require children to mentally impose boundaries and to relate each region to a numeral on its periphery. Even with these challenges, the idea that the clock hand indicates the hour by containment within a region should carry over. These are precisely the kinds of embodied experiences a teacher can draw upon to help students learn to conceive of a new situation in relevant ways, but only if the teacher knows that learners have had these kinds of experiences and senses the commonality in image-schematic structure that would make one experience a useful resource for understanding another. Where such experiences are lacking, a dynamic animation could depict the desired

Changes in Understanding

The study also demonstrates how those who perform capably can harbor quite different levels of understanding and how these understandings can change unnoticeably. Students can learn to perform the procedure for borrowing and regrouping when solving two-digit subtraction problems before they develop much understanding of the place-value system, just as they can learn to name relative times like "quarter past" with little awareness of the connection between the movements of the long and short hands. Adding image-schematic structure to a conceptualization provides an opportunity for insight: for discovering conceptual links and relations that can support reasoning in novel situations. A firm grasp of the PART-WHOLE structure of place-value arithmetic, for example, enables the learner to adapt borrowing and regrouping to solve two-digit subtraction problems in base-7, just as grasping the complex PART-WHOLE and SOURCE-PATH-GOAL relations of clock times equips the learner to interpret novel or foreign time statements such as "quarter past ten thirty" or "five before half ten." Rich image-schematic structure carries the learner beyond familiar contexts, supporting transfer of learning to new situations.

Applying Cognitive Semantics to the Study of Mathematical Learning

These considerations about the nature of mathematical conceptualization illuminate a basic tension in instruction. On the one hand, instruction is highly situated, with the choice of instructional maneuvers depending crucially on the particulars of the situation, the learners, their history, and how they conceptualize what they are confronting; on the other hand, there are familiar progressions of learning with known obstacles. With regard to the latter, there may be value in bringing advances in cognitive semantics—in understanding the conceptual structures and operations that humans use to make sense of language and the world—to bear on thorny problems of mathematics teaching and learning. Applying cognitive semantics to the study of classroom discourse can reveal variations in the way mathematical ideas are conceptualized, especially at different stages of learning, providing possibilities for more informed instruction. In doing so, we need to pay attention to how prior learning structures the way learners conceptualize new experiences in ways that both facilitate and interfere with proper understanding. I have purposefully chosen the everyday practice of time-telling to illustrate the cognitive complexities involved in seemingly simple activities and the insights that might be gained from such study.

Pioneering work in the cognitive semantics of mathematics is Lakoff and Núñez's (2000) examination of the image schemas, conceptual metaphors (projections of image-schematic structure), and conceptual blends (integrations of compatible image-schematic structure) that underlie many of our most fundamental mathematical ideas.[8] Their

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image-schematic structure, helping the learner to see the interrelations and to conceptualize the situation in this productive way.

[8] An example of a conceptual metaphor is the set of correspondences linking object collections to arithmetic discussed earlier. Additional conceptual metaphors link arithmetic with geometry, and so on, providing image-schematic structure for conceptualizing abstract mathematical ideas (see Lakoff and Núñez, 2000, for many examples and discussion). Examples of space-number conceptual blends include the number line and complex numbers, which integrate two-dimensional space with real/imaginary numbers (discussed in Fauconnier and Turner, 2002, pp. 270-274, and in Lakoff & Núñez, 2000, pp. 420-

analysis of the discretization program in mathematics—the attempt to base all of mathematics on set theory and logic—illustrates the distinctions between the static conceptualizations implicit in formal set-theoretic definitions (for example, that a line segment is a set of points, or that a function is a mapping between *x* and *y*) and the dynamic conceptualizations that learners often find more intuitive (for example, that a line segment is the path defined by a moving point, or that *y* increases as *x* increases)—and that teachers, despite their commitment to proper discretized mathematics, nevertheless employ when teaching mathematical ideas (see, e.g., Núñez, 2007). We see something of the flavor of this static/dynamic divide in the conceptualization of "quarter past" as naming the upper right quarter-circle versus as naming the portion of the path already traversed by the long hand in its journey through the hour. For teachers and learners of mathematics, it is not so much a matter of deciding which conceptualization is the right one (except perhaps for working out the details of a formal proof) as it is a matter of recognizing that there are different ways of conceptualizing mathematical ideas and that each makes some things apparent while rendering others invisible. Part-whole conceptualizations support some inferences while motion-based conceptualizations support others. Often the two are complementary, yielding different insights to the mathematical thinker and making both an integral part of the mathematics curriculum. A greater sensitivity to image-schematic structure could help us be more aware of these conceptual shifts and contrasts.

Implications for Teaching Mathematics

Teachers of mathematics can take several lessons from the study. One is to bear in mind that learners may perform successfully using conceptualizations that are highly impoverished or even erroneous. Despite the best attempts to help learners understand the meaning of "a quarter past," they may take away simply the idea that you say "a quarter past" when the long hand points at the *3*. That is certainly enough to get them started. The real understanding may emerge later, after an extended period of successful performance, and actually performing the task is more likely to generate that understanding. Another lesson is not to be too disheartened by sudden breakdowns but instead to see them as opportunities to probe for latent errors that have gone unnoticed. Seeking explanations for correct responses and posing unusual problems are two ways to probe for latent errors. Another is to have students take on the role of teacher and then to pay attention not only to what they say but also to where they look and how they gesture as clues to how they are conceptualizing the situation. The teaching role also gives students an opportunity to discover conceptual relations in their own explanations and thereby to enrich their understanding. Yet another lesson is to be sensitive to order: to what students have been told and what they have been doing, to how this might determine how they will conceptualize a new problem situation, and to how that new situation might demand a shift in their conceptualization such as a different way of conceiving of the short hand's role in reading the hour. Focusing on the conceptual challenges inherent in new learning from the learner's point of view can lead to more proactive teaching, for

432). It's important to bear in mind that the issue here is not how mathematical ideas are formally defined but how they are conceptualized—how the human mind is able to think mathematically and to learn and understand mathematical ideas.

example, to deliberately emphasizing the relation between long and short hand motion before teaching relative time so that students will be more likely to link source to source and goal to goal. All of this advice describes what good teachers already do; what the image schema analysis adds is a new way of thinking about what is going on beneath the surface, a way that informs a more diagnostic kind of teaching.

Implications for Research on Understanding and Adaptive Expertise

For the learning sciences, the analysis presented here has implications for how we view the nature of understanding and of adaptive expertise. The focus on image-schematic structure occupies a middle ground between two influential views of understanding, one emphasizing mental models (Gentner & Stevens, 1983) and the other emphasizing a "flexible performance capability" (Perkins, 1998). Perkins criticizes the mental models view, arguing that learners may have the proper mental model yet not be able to operate with it and saying that many understanding performances, such as using language grammatically, do not seem to involve explicit mental models at all. Perkins' preferred definition of understanding as "the ability to think and act flexibly with what one knows" (p. 40) begs the question as to *how* one is able to think and act flexibly—on what basis? Perkins chooses to remain agnostic as to the basis and to focus instead on how a performance view of understanding impacts teaching and learning practices. This approach is productive as far as it goes, but learning scientists would still like to know what makes such performances possible.

From the perspective of embodied cognition, the image-schematic structure of conceptualizations appears to be at least part of the answer. The analysis of image schemas in clock-reading demonstrates a basis for flexible performance that does not demand an explicit mental model. The conceptualizations involved in reading the hour for different clock times are not explicit mental models in the sense of Gentner and Stevens (1983), such as imagining the operation of an electrical circuit as the flow of liquid through wires; they are instead more like grammatical constructions in that learners operate with them without consciously attending to the image-schematic structure that underlies their thinking. While conceptualizations can certainly involve imagery and mental simulation, such as imagining a clock hand moving to or from a reference point, the image-schematic structure that underlies this simulation (SOURCE-PATH-GOAL, PART-WHOLE, etc.)—and that gives it its explanatory power—remains implicit. The example of emerging expertise in reading "a quarter past" also demonstrates how a learner can become able to act more powerfully and flexibly through enrichment of image-schematic structure rather than through a shift to a different mental model. In some cases, shifting to a different conceptualization may be exactly what is called for, such as switching from an object collection to a number line view of subtraction; here the learner does not need to consciously model removing objects from a collection or moving to the left along a line, only to operate with the image-schematic structure that derives from these different kinds of experiences. The image schema analysis even helps to explain why explicit mental models can be effective: imagining liquid flowing through wires is powerful precisely because it provides the image-schematic structure needed to support certain inferences. Image schemas provide the relational and dynamic structure that enables mental models to do their work, yet they also enable us to think without vivid imagery using a retained or emerging sketchy sense of the interrelations among elements.

The same arguments provide at least a partial account for the kind of creative adaptability that characterizes adaptive expertise (Hatano & Inagaki, 1986; Bransford, Brown, & Cocking, 1999; Schwartz, Bransford, & Sears, 2005). Here, as in Lakoff and Núñez's analysis of mathematics, conceptual metaphor and blending could play a role in linking diverse experiences—projecting, merging, and enriching image-schematic structure—to provide a more powerful basis for engaging new situations.

CONCLUSION

This brief examination of image schemas in clock-reading has provided insight into aspects of learning that remain largely imperceptible: the bodily basis of how learners form conceptual understandings, the persistence of latent errors into later stages of learning, and the nature of conceptual changes that enable more expert-like performance. Learners are likely to operate with impoverished and even erroneous conceptualizations as they gain familiarity with new topics in mathematics, and these can lead to sudden breakdowns or disruptions of future learning. Sensitivity to the image-schematic structure of different ways of thinking about problems, to the learning histories of individuals and the conceptual resources they bring to bear, and to the conceptual shifts required to tackle new classes of problems can help teachers anticipate difficulties, guide learners toward more appropriate or enriched conceptualizations, and probe for the reasoning and depth of understanding behind learners' conclusions. For the learning sciences, attention to the image-schematic structure of conceptualizations helps explain the efficacy of mental models and the conceptual basis for flexible performance, relating both to embodied aspects of human cognition. With respect to the topic of this special issue, studies of how teachers and learners use their bodies when they think and communicate mathematically are likely to gain explanatory power when they relate observable bodily actions to the image-schematic structure and dynamics of ongoing conceptualization.

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TABLES

TABLE 1
Comparison of Landmark, Relative, and Absolute Times on the Analog Clock.

	Landmark	*Relative*	*Absolute*
Time format	<H_{LM}> o’clock	<Q/M_{int}> past/till <H_{ref}>	<H_{cur}> <M_{cur}>
Order of looking at dials	Hour	Quarter-hour *or* minute Hour	Hour Minute
Image schema for hour	PROXIMITY	SOURCE-PATH-GOAL	CONTAINER

TABLE 2
Percentage of Times Named Correctly in Studies of American Children’s Clock-Reading (compiled from Springer, 1952; Friedman & Laycock, 1989; Siegler & McGilly, 1989)

<table>
<tr><th></th><th colspan="5">Type of Time</th></tr>
<tr><th>Age</th><th>Hour (3:00)</th><th>30-min. (3:30)</th><th>15-min. (3:15/45)</th><th>5-min. (3:25)</th><th>1-min. (3:27)</th></tr>
<tr><td>4</td><td>10%</td><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td>45%</td><td>10%</td><td>5%</td><td></td><td></td></tr>
<tr><td>6</td><td>75%</td><td>30%</td><td>15%</td><td></td><td>5%</td></tr>
<tr><td>7</td><td>100%</td><td>90%</td><td></td><td></td><td>35%</td></tr>
<tr><td>8</td><td></td><td>95%</td><td colspan="2" rowspan="2">80%</td><td>60%</td></tr>
<tr><td>9</td><td></td><td>100%</td><td>70%</td></tr>
<tr><td>10</td><td></td><td></td><td></td><td></td><td>80%</td></tr>
</table>

TABLE 3
Correct Naming of Times Using Different Image Schemas to Read the Hour

	Image Schema		
Type of Time	*PROXIMITY*	*CONTAINER*	*SOURCE-PATH-GOAL*
Absolute			
Long hand right (H:15)	Yes (latent error)	Yes	
Long hand left (H:45)	No (error)	Yes	
Relative			
Long hand right (qtr past H)	Yes (latent error)	Yes (latent error)	Yes
Long hand left (qtr till H)	Yes (latent error)	No (error)	Yes

FIGURES

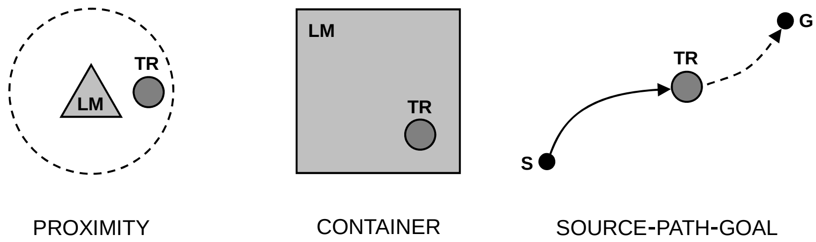

FIGURE 1 Three image schemas involved in reading the hour on an analog clock.

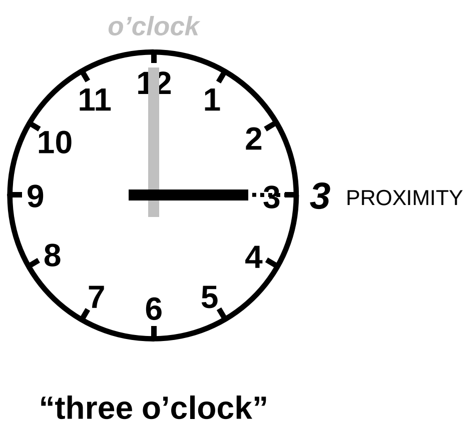

FIGURE 2 Reading landmark times as "<number> o'clock."

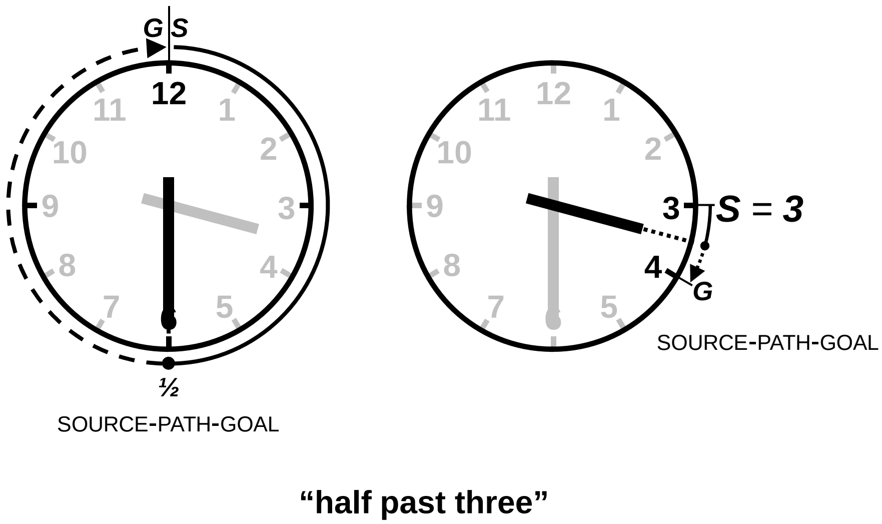

FIGURE 3 Reading relative time (time past or till the hour).

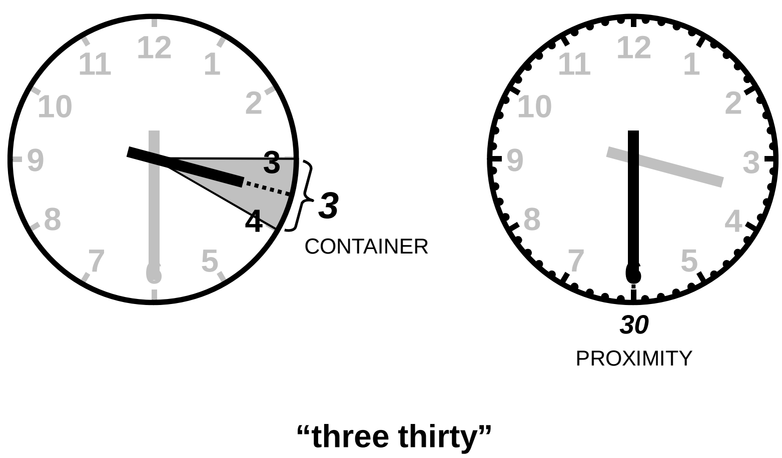

FIGURE 4 Reading absolute time (in hours and minutes).

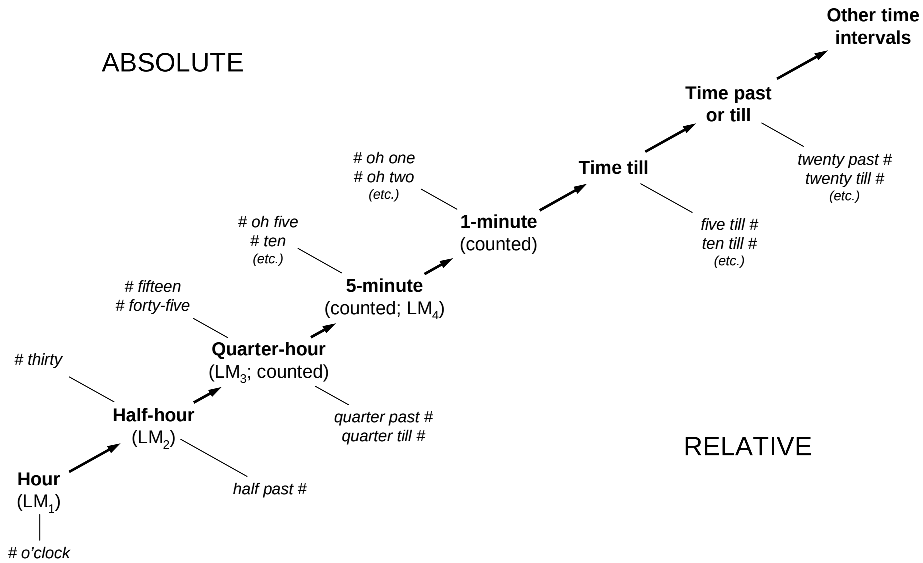

FIGURE 5 Progression of learning to read times on an analog clock

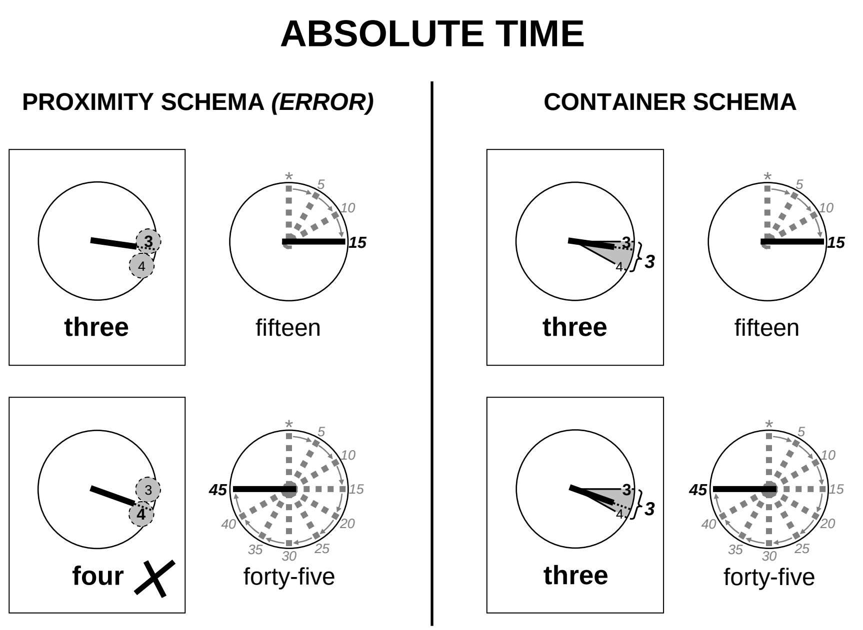

FIGURE 6 Overgeneralization of the PROXIMITY image schema from landmark to absolute time. The use of the proper image schema is shown on the right.

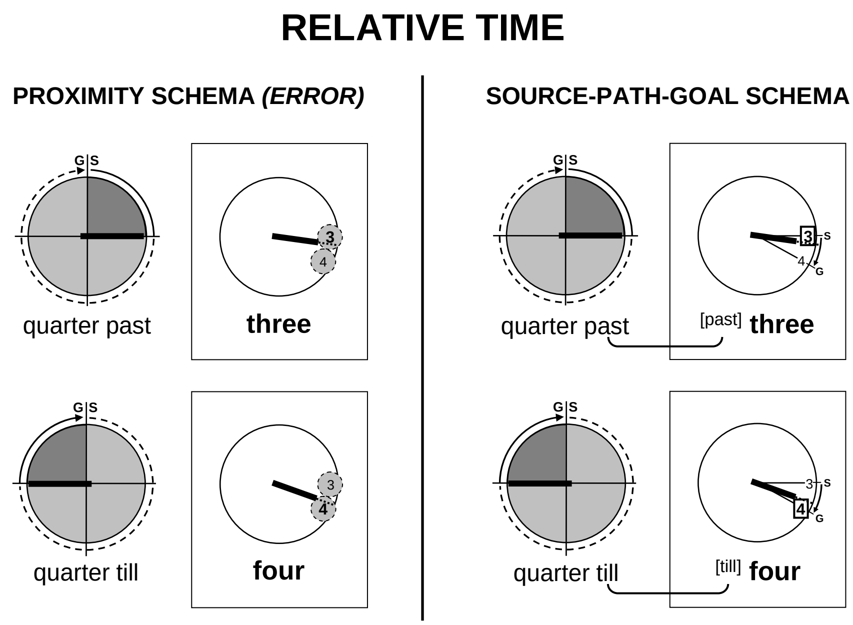

FIGURE 7 Overgeneralization of the PROXIMITY image schema from landmark to relative time. The use of the proper image schema is shown on the right.

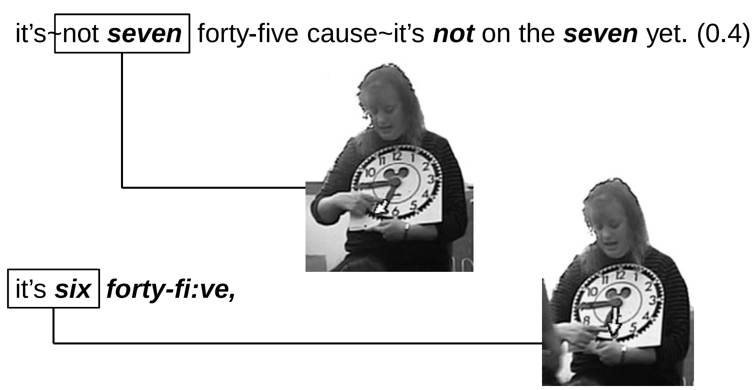

FIGURE 8 Excerpt from 1st grade instruction on reading "six forty-five."

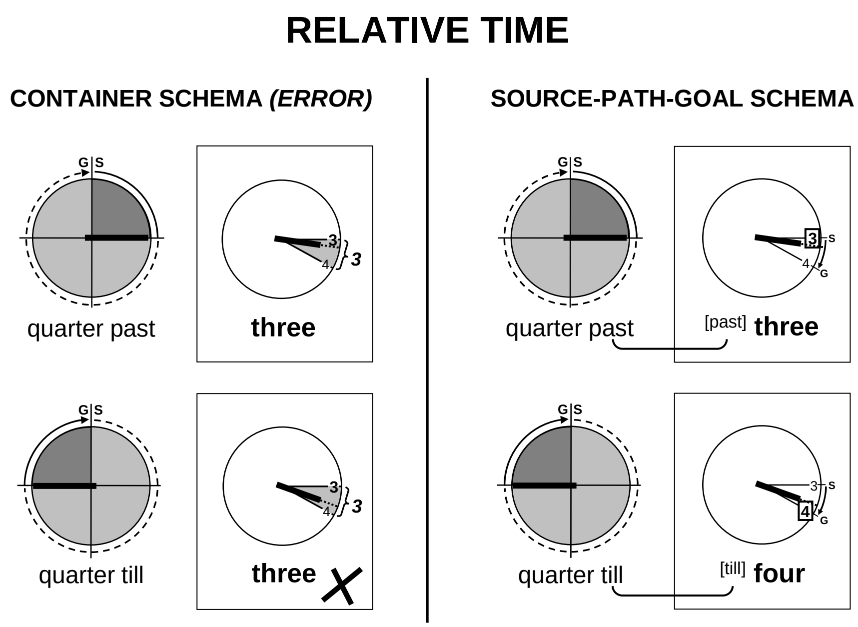

FIGURE 9 Overgeneralization of the CONTAINER image schema from absolute to relative time. The use of the proper image schema is shown on the right.

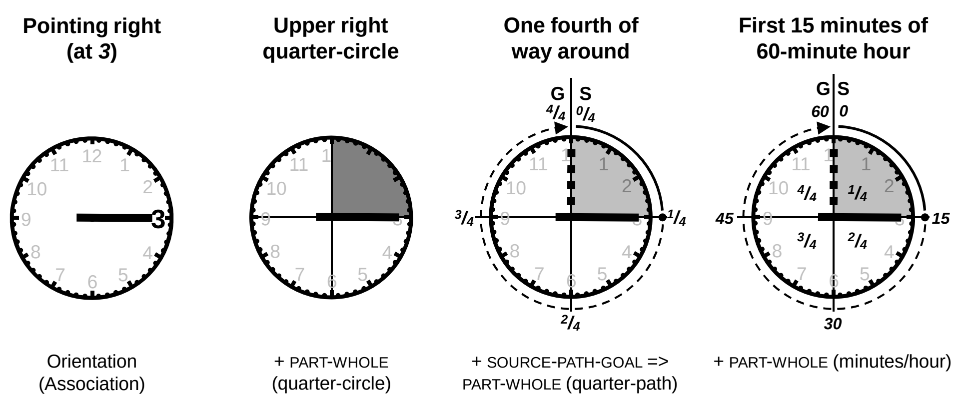

FIGURE 10 Emerging expertise in conceptualizing “a quarter past.” Conceptualizations to the right have richer image-schematic structure.